

T/E



Industria y Comercio
SUPERINTENDENCIA

SOLICITUD
PATENTE DE INVENCION
PCT

EXPEDIENTE N°

06-54051

TITULO ESTIMADOR DE CANAL CONJUGADO Y EFICIENTE BASADO EN GRADACIONES

CLASIFICACION INTERNACIONAL _____

SOLICITANTE ZENITH ELECTRONICS
CORPORATIONS

DOMICILIO ILLINOIS, ESTADOS UNIDOS DE AMERICA

APODERADO DARIO CARDENAS NAVAS

BOGOTA, D C _____

PETITORIO

Industria y Comercio
SUPERINTENDENCIA

SUPERINTENDENCIA DE INDUSTRIA Y COMERCIO
Rad: 06-054051- -00000-0000
Fec: 2006-06-05 17:28:36 Dep. 2020 NUECREACIONES
Tra. 11 PATENTEN
Eve: 370 FASENACIONALM
Act 411 PRESENTACION Folios: 200

FORMULARIO UNICO DE SOLICITUD DE PATENTE PCT - SOLICITUD DIVISIONAL ENTRADA EN FASE NACIONAL

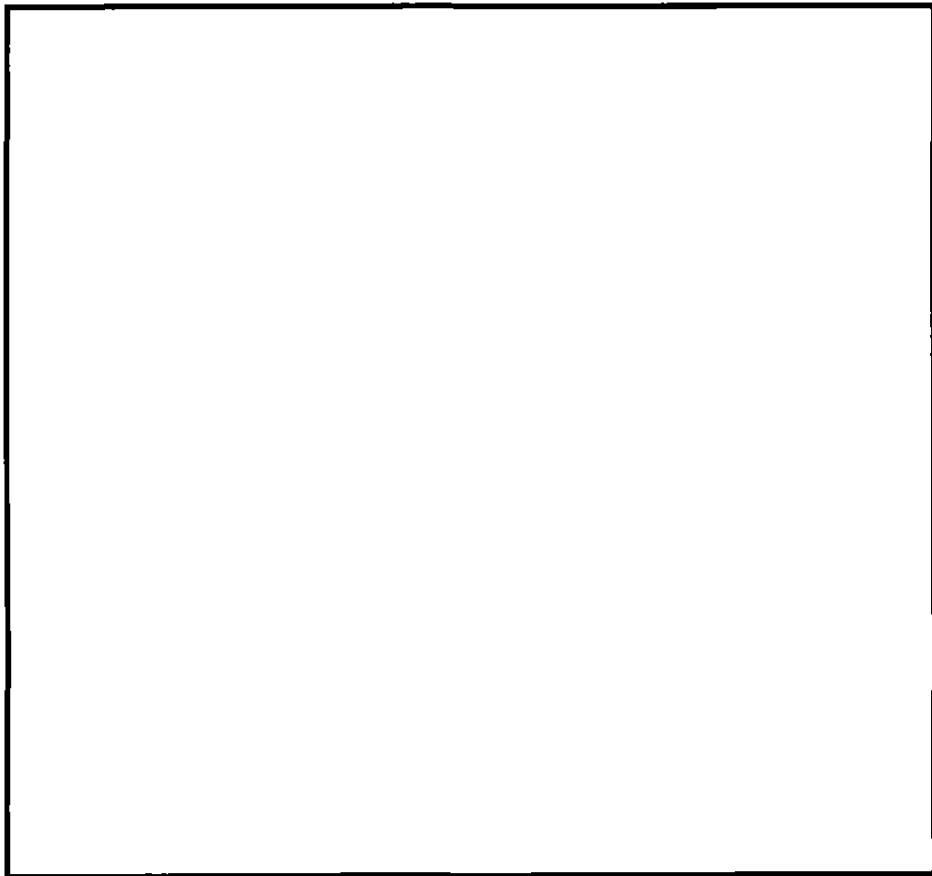
① SOLICITUD DE: ☒ Patente de Invención ☐ Patente de Modelo de Utilidad ☐ Capítulo I ☒ Capítulo II			
② S LICITANTE (71)	Nombre: ZENITH ELECTRONICS CORPORATION Dirección: 2000 Millbrook Drive, Lincolnshire, Illinois 60069, Estados Unidos de América Nacionalidad o Domicilio: Estado de Illinois, Estados Unidos de América Lugar de Constitución: Estado de Illinois, Estados Unidos de América Teléfono: 3123600 Fax: 3122410 E-mail: <u>general@cardenasycardenas.com</u>	IDENTIFICACION C.C. ☐ NIT ☐ C.E. ☐ Otro ☒ Cual _____ Número _____	
③ REPRESENTANTE O APODERADO	Nombre: DARIO CARDENAS NAVAS Dirección: Carrera 7 No 71-52 Torre B Piso 9 Teléfono: 3123600 Fax: 3122410 E-Mail: <u>general@cardenasycardenas.com</u>	IDENTIFICACION C.C. ☒ NIT ☐ C.E. ☐ Otro ☐ Cual _____ No 17.066.629 TP 1.250	
④ INVENTOR(ES) (72)	Nombre: 1. Jin H. Kim 2. Serdar Ozen Dirección: 1. 550 Timberwood Drive, Lake Zurich, IL 60047, Estados Unidos de América 174 Hemingway Court, Vernon Hillis, IL 60061, Estados Unidos de América Nacionalidad o Domicilio: 1. Illinois, Estados Unidos de América 2. Illinois, Estados Unidos de América		
⑤ PCT (86) Solicitud Internacional No. <u>PCT/US2004/034842</u> Fecha <u>Octubre 18, 2004</u> Publicación Internacional No. <u>WO 2005/082582 A1</u> Fecha <u>Julio 7, 2005</u>			
⑥ Título(54): ESTIMADOR DE CANAL CONJUGADO BASADO EN GRADACIONES			
⑦ Clasificación Internacional (51):			
⑧ Prioridad SI ☒ N ☐	(33)País de Origen <u>U.S.A.</u>	(32)Fecha <u>Diciembre 5, 2003</u>	(31)N° de Solicitud <u>10/729,722</u>
⑨ Comprobante de Pago No.: <u>06-42,489,</u> Fecha <u>JUNIO 5/2006.</u>			

10

ANEXOS

- ☒ Comprobante de pago de la tasa de presentación de la solicitud. N° 06-42,489-
- ☒ Documento que acredita la existencia y representación legal cuando el solicitante sea persona jurídica.
- ☒ Poderes, si fuere el caso. (Ya reposa en el expediente)
- ☐ Documento de cesión del inventor al solicitante o a su causante.
- ☐ Declaraciones
- ☒ Copia de la solicitud internacional. (Resumen, descripción, reivindicaciones, dibujos)
- ☒ Traducción de la solicitud internacional al castellano. (Resumen, descripción, reivindicaciones, dibujos)
- ☒ Copia del examen preliminar internacional efectuado por la Autoridad Internacional de Examen Preliminar (IPEA).
- ☐ Traducción del examen preliminar internacional efectuado por la IPEA.
- ☐ De ser el caso, copia del contrato de acceso.
- ☐ De ser el caso, documento que acredita la licencia o autorización de uso de conocimientos tradicionales de las comunidades indígenas.
- ☐ De ser el caso, certificado de depósito del material biológico.
- ☐ Arte final 12x12.
- ☐ De ser el caso, información sobre otras solicitudes de patente o títulos obtenidos en el extranjero por el mismo titular o su causante, relacionadas parcial o totalmente con la invención de esta solicitud.
- ☐ De ser el caso, modificaciones efectuadas a la solicitud internacional.
- ☐ Otros.

11 FIGURA CARACTERISTICA



12

Solicito la Concesión de la patente,

NOMBRE: DARI CARDENAS NAVAS

FIRMA:

C.C. 17.066.629 TP. 1.250

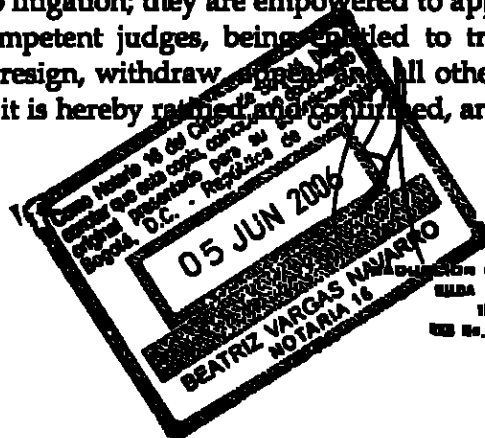
- Colombia -

POWER OF ATTORNEY

In the city of Lincolnshire, on the 9th day of the month of October, of the year two thousand and two (2.002), before me, Sharon K. Kroll, Notary Public, appeared Jack Kail, of 57 age, and domiciled at Highland Park, Illinois, and stated:

FIRST: That he/she acts in his/ her capacity of Assistant Secretary of Zenith Electronics Corporation, a company legally constituted and in existence in accordance with the laws of the state of Illinois, and domiciled at 2000 Millbrook Drive, Lincolnshire, Illinois 60069, United States of America, as evidenced in the By-laws and Articles of Incorporation, documents which I have examined.

SECOND: That in such capacity he/she grants hereby special, ample and sufficient Power of Attorney to DARIO CÁRDENAS NAVAS and/or EDUARDO CÁRDENAS CABALLERO, and/or BERNARDO P. CÁRDENAS MARTINEZ and/or JAMES VALDIRI REYES and/or LUZ HELENA ADARVE GÓMEZ, and/or JUANITA ACOSTA GÓMEZ and/or ANA MARÍA RUIZ RUEDA, registered lawyers of the firm CÁRDENAS & CÁRDENAS PROPIEDAD INTELECTUAL LAWYERS, domiciled at Carrera 7 No. 71-52, Torre B, Piso 9 Bogotá, D.C. Colombia, to carry before the corresponding Colombian national offices and authorities the proceedings to obtain patent, industrial models and designs and trademark registrations, as well as proceedings of renewals, assignments, change of name or domicile of the owner, waivers and licenses of the same for which purpose they are empowered to take before said authorities all necessary actions for the indicated purposes; to file applications, statements, claims, formulate descriptions, amendments, oppositions and appeals; pay all taxes and installments and make any other payments determined by law; to receive every document and values issuing the respective discharge; to fulfill any other requirements and take, in general, every measure that they shall deem appropriate for the defense and protection of the trademarks of the company in the event of infringements, oppositions, proceedings for cancellation or annulment, imposition of fines, including resort to litigation; they are empowered to appear as plaintiff or defendant before the competent judges, being entitled to transact, submit to arbitration, waive, receive, resign, withdraw and perform all other legal powers which may be necessary, and it is hereby ratified and confirmed, any such



ADJUNCIÓN OFICIAL NO.
SILDA AMULENA DE PIDEANITA
INTERPRETE OFI.-L
NO. 14, 202 DEL MUN. DE JANTEN

act said attorney lawfully do for its benefit, as my own act granting them also power to substitute this proxy and if necessary to revoke such substitutions.

THIRD: That for purposes of articles 597 and 543 of the Colombian code of Commerce the attorneys are empowered to receive service of notice and designate judicial and extra judicial attorneys.

SIGNED AT Lincolnshire, Illinois, U.S.A.

ON October 9, 2002

BY *Jack Kail*
Jack Kail

Notary Public



AGUACION OFICIAL DEL
MILIA AGUILERA DE PRIMAVERA
INTERPRETE OFICIAL
EN LA, 777 DEL MUN. DE ...



SPRINGFIELD, ILLINOIS

APOSTILLE

(Convention de La Haye du 5 Octobre 1961)

1. Country: United States of America
This public document
2. has been signed by SHARON K. KROLL
3. acting in the capacity of NOTARY PUBLIC, COOK COUNTY
4. bears the seal/stamp of STATE OF ILLINOIS

Certified

5. Springfield, Illinois
6. OCTOBER 11, 2002
7. by the Secretary of State, State of Illinois
8. No. 5401
9. Seal/Stamp:

10. Signature:

Jesse White

JESSE WHITE
SECRETARY OF STATE
STATE OF ILLINOIS



6

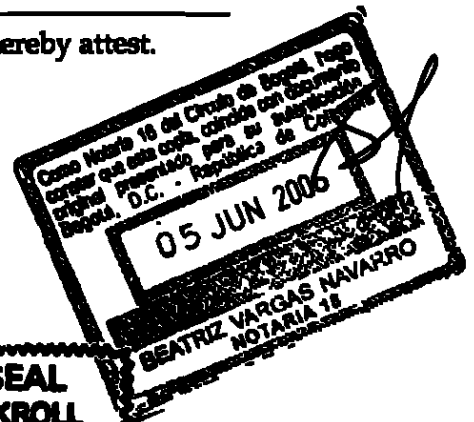
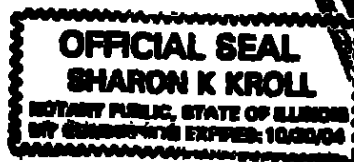
NOTARIAL CERTIFICATION

The undersigned Notary Public certifies that Mr. Jack Kail appeared before me and signed the preceding document; that I know him personally and I am certain that he has legal capacity to grant it; that he is Assistant Secretary of Zenith Electronics Corporation, and is consequently authorized to grant such document; that such company is organized under the laws of the state of Illinois, and is domiciled at 2000 Millbrook Drive, Lincolnshire, Illinois 60069, United States of America; and that the act for which this document is granted is included among those which constitute the corporate object of the named company. Furthermore, that such company performs its corporate purpose in accordance with the laws of its incorporation and domicile.

I am certain of all these facts because I have examined the respective documents consisting of Colombian Power of Attorney

all of which I hereby attest.

NOTARY PUBLIC (signature) Sharon K Kroll
ISSUED AND SIGNED AT Lincolnshire, Illinois
DATED October 9, 2002



(LEGALIZATION BY APOSTILLE IS REQUIRED)

REAGUACION ORIGINAL DEL
CIRCUITO DE BOGOTÁ
AUTENTICACIÓN DEL
DOCUMENTO ORIGINAL

Traducción Oficial No. 2002/195 de un documento escrito en inglés el cual para su identificación se sella con el sello del traductor al tiempo con la presente. Esta traducción ha sido preparada por Hilda Aguilera de Piedrahita, traductora oficial debidamente reconocida por el Ministerio de Relaciones Exteriores y autorizada por el Ministerio de Justicia y del Derecho por Resolución No. 323 de 1980.

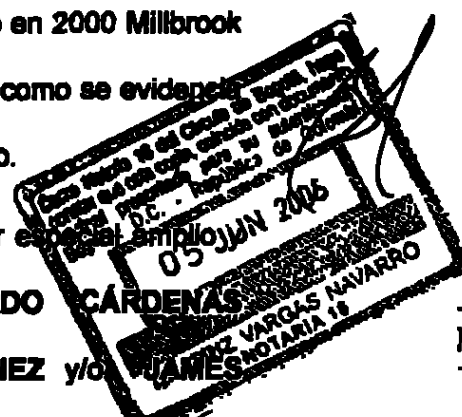
PODER

Colombia

En la ciudad de Lincolnshire, el día 9 del mes de Octubre de dos mil dos(2002), ante mí, Sharon K. Kroll Notario Público, compareció Jack Kail de 57 años de edad, y con domicilio en Highland Park, Illinois y manifestó:

PRIMERO: Que el obra en su calidad de Secretario Asistente de Zenith Electronics Corporation, una compañía legalmente constituida y existente de conformidad con las leyes del estado de Illinois, y con domicilio en 2000 Millbrook Drive, Lincolnshire, Illinois 60069, Estados Unidos de América, como se evidencia en los Estatutos de Constitución, documentos que he examinado.

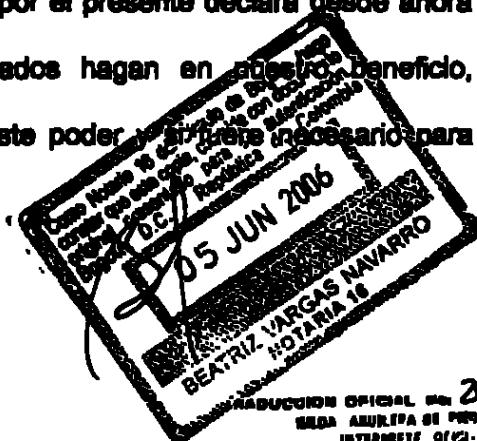
SEGUNDO: que en tal calidad el otorga por el presente poder especial suficiente a **DARÍO CÁRDENAS-NAVAS y/o EDUARDO CÁRDENAS CABALLERO, , y/o BERNARDO P. CARDENAS MARTINEZ y/o VALDIRI REYES y/o LUZ HELENA ADARVE GOMEZ y/o JUANITA AGOSTA GÓMEZ y/o ANA MARÍA RUIZ RUEDA**, abogados registrados de la firma **CÁRDENAS & CÁRDENAS ABOGADOS PROPIEDAD INTELECTUAL LTDA.**



TRADUCCION OFICIAL No. 2002/195
HILDA AGUILERA DE PIEDRAHITA
INTERPRETE OFICIAL
MINISTERIO DE JUSTICIA Y DEL DERECHO

9

domiciliada en la Carrera 7 No. 71-52, Torre B, Piso 9, Bogotá D.C., Colombia, para llevar a cabo ante las correspondientes Oficinas Nacionales Colombianas y autoridades los trámites para obtener patentes, modelos industriales y diseños y registros de marcas, así como trámites para renovaciones, traspasos, cambios de nombre o domicilio del propietario, renunciaciones y licencias de los mismos para cuyo fin están facultados para realizar ante dichas autoridades todas las acciones necesarias para los fines indicados; para presentar solicitudes, declaraciones, reivindicaciones, formular descripciones, modificaciones, oposiciones, y apelaciones; pagar todos los impuestos y derechos y hacer cualesquiera otros pagos que la ley determine; recibir todos los documentos y valores con expedición del respectivo descargo; cumplir cualesquiera otros requisitos y tomar en general todas las medidas que juzguen oportunas para la defensa y protección de las marcas de la compañía en el evento de infracciones, oposiciones, procesos de cancelación o anulación, imposición de multas, presentar antecedentes ante las cortes; quedan facultados para comparecer en calidad de demandante o demandado ante los jueces competentes, tendrán derecho de transar, someterse a arbitraje, renunciar, recibir, renunciar, desistir, apelar y todas las demás facultades legales que puedan requerirse, y por el presente declara desde ahora válido y satisfactorio lo que dichos abogados hagan en su propio beneficio, otorgándoles también poder para sustituir este poder y para todo lo necesario para revocar tales sustituciones.



2002/195
RADUCCION OFICIAL No.
SALA AUXILIAR DE PROTESTAS
INTERPRETE OFICIAL
VIRGILIO, VIRGILIO DEL MAR. 01 01...

TERCERO: que para fines de los artículos 597 y 543 del Código Colombiano de Comercio, los abogados están facultados para recibir notificaciones y para designar apoderados judiciales y extrajudiciales.

FIRMADO EN: Lincolnshire, Illinois, EUA

EL: 9 de Octubre de 2002.

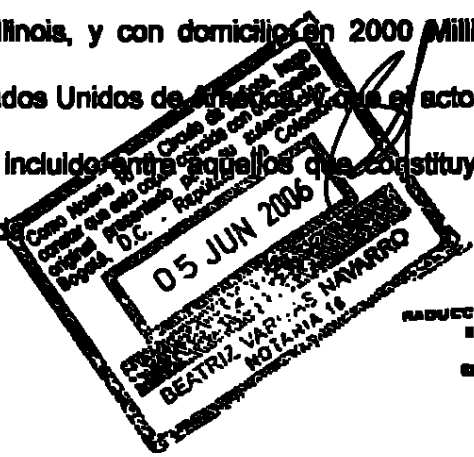
POR: Illegible

Jack Kail

NOTARIO PUBLICO

CERTIFICACIÓN NOTARIAL

El suscrito Notario Público certifica que el Sr. Jack Kail compareció ante mí y suscribió el documento precedente; que lo conozco personalmente y que estoy seguro de que tiene las facultades para otorgarlo; que él es Secretario Asistente de Zenith Electronics Corporation, y por consiguiente está autorizado para otorgar dicho documento; que la compañía mencionada existe y está constituida bajo las leyes del estado de Illinois, y con domicilio en 2000 Millbrook Drive, Lincolnshire, Illinois, 60069, Estados Unidos de América, que el acto para el cual se otorga este documento está incluido entre aquellos que constituyen el objeto social de la compañía mencionada.



REDUCCION OFICIAL No. 2002/195
HILDA AGUILERA DE PINO
INTERPRETE OFICIAL
CRA No. 322 DEL MAN. DE BOGOTÁ

Estoy seguro de todos estos hechos por haber examinado los documentos respectivos consistentes en el Poder para Colombia, de todo lo cual doy fe.

NOTARIO PUBLICO (Firma) (Firmado:Sharon K. Kroll

EXPEDIDO Y FIRMADO EN Lincolnshire, Illinois

FECHA : Octubre 9, 2002.

Sello a tinta: SELLO OFICIAL, SHARON K. KROLL, Notario Público, Estado de Illinois,

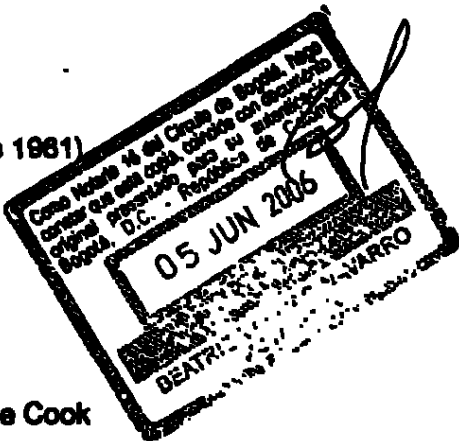
Mi comisión expira 10/30/04.

(SE REQUIERE LEGALIZACIÓN MEDIANTE APOSTILLA)

ESTADO DE ILLINOIS
SECRETARIO DE ESTADO
SPRINGFIELD, ILLINOIS
APOSTILLA

(Convención de la Haya de Octubre 5 de 1981)

1. País: Estados Unidos de América
Este documento público
2. Ha sido firmado por SHARON K. KROLL
3. Actuando en calidad de Notario Público, Condado de Cook
4. Lleva el sello/estampilla de l Estado de Illinois



CERTIFICADO

ADUCCION ORIGINAL, 2002/195
LUNA, JUAN CARLOS
INTERPRETE
2002/195, LUNA, JUAN CARLOS

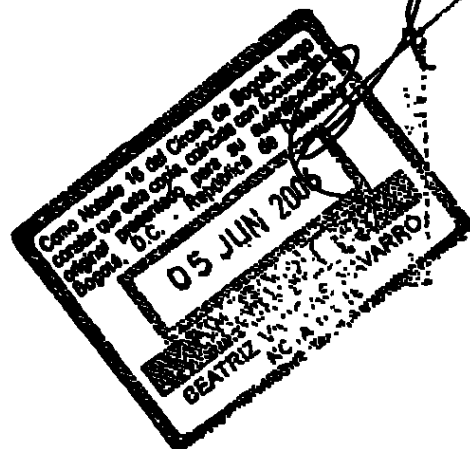
5. **En Springfield, Illinois**
6. **El día 11 de Octubre de 2002.**
7. **Por el Secretario de Estado del Estado de Illinois**
8. **Número 5401.**
9. **Sello (Sello Redondo en Relieve)**
10. **Firma (Firmado: Ilegible)**

Jesse White, Secretario de Estado, Estado de Illinois.

ES TRADUCCIÓN FIEL Y COMPLETA de la cual se deja copie en los archivos de esta oficina para su confrontación y a la cual me remito.

Bogotá, Octubre 21 de 2002.

2002/195



MINISTERIO DE RELACIONES
EXTERIORES

Lucy Ramirez

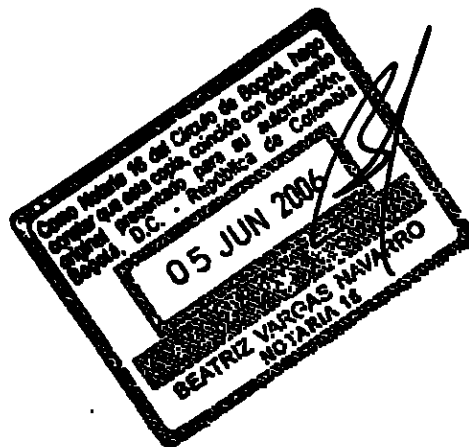
023896

NO SE ASUME
RESPONSABILIDAD DEL TEXTO

2002 OCT 22 P 3 00

CUYA FIRMA APARECE EN EL
DOCUMENTO QUE ANHEA
LAS FIRMAS DE LOS

BBR
JEFE DE LEGALIZACIONES
SANTAFE DE BOGOTA D.C.



E-mail del 5 de Junio de 2.006
lsdavar@cal2.vsnl.net.in

Ref: Bajaj Auto Limited - Colombia - Industrial Design application entitled
"Motorcycle". File No. 04-114438. Our ref: D-023.

Dear Sirs:

Thank you for your e-mail dated June 2, 2.006 in regards to the above matter.

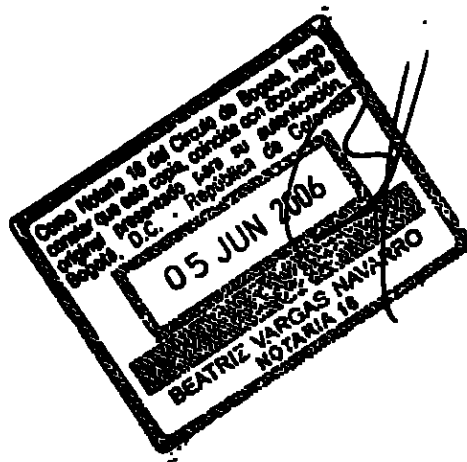
Kindly note that since the reconsideration and appeal brief filed against the rejection of the referenced industrial design application has not yet been decided by the Colombian Patent Office, no maintenance fees have to be paid up this moment.

We will keep you informed about any development regarding this matter.

Yours sincerely,

LUZ HELENA ADARVE-GÓMEZ
AMR/lu/Varios/Diversos/Bajaj Auto Limited, Inc./
F & DMS

ANA MARIA RUIZ-RUEDA



PCT REQUEST

Original (for SUBMISSION)

0	For receiving Office use only	
0-1	International Application No.	
0-2	International Filing Date	
0-3	Name of receiving Office and "PCT International Application"	
0-4	Form PCT/RO/101 PCT Request	
0-4-1	Prepared Using	PCT-SAFE [EASY mode] Version 3.50 (Build 0002.163)
0-5	Petition The undersigned requests that the present international application be processed according to the Patent Cooperation Treaty	
0-6	Receiving Office (specified by the applicant)	United States Patent and Trademark Office (USPTO) (RO/US)
0-7	Applicant's or agent's file reference	FP03,0369-WO
I	Title of invention	EFFICIENT CONJUGATE GRADIENT BASED CHANNEL ESTIMATOR
II	Applicant	
II-1	This person is	applicant only
II-2	Applicant for	all designated States except US
II-4	Name	ZENITH ELECTRONICS CORPORATION
II-5	Address	2000 Millbrook Drive Lincolnshire, Illinois 60069 United States of America
II-6	State of nationality	US
II-7	State of residence	US
II-8	Telephone No.	847 941-8011
II-9	Facsimile No.	847 941-8003
III-1	Applicant and/or inventor	
III-1-1	This person is	applicant and inventor ✓
III-1-2	Applicant for	US only
III-1-4	Name (LAST, First)	KIM, Jin H.
III-1-5	Address	550 Timberwood Drive Lake Zurich, Illinois 60047 United States of America
III-1-6	State of nationality	US
III-1-7	State of residence	US

PCT REQUEST

Original (for SUBMISSION)

III-2	Applicant and/or inventor	
III-2-1	This person is	applicant and inventor ✓
III-2-2	Applicant for	US only
III-2-4	Name (LAST, First)	OZEN, Serdar
III-2-5	Address	174 Hemingway Court Vernon Hills, Illinois 60061 United States of America
III-2-6	State of nationality	TR
III-2-7	State of residence	US
IV-1	Agent or common representative; or address for correspondence The person identified below is hereby/ has been appointed to act on behalf of the applicant(s) before the competent International Authorities as:	agent
IV-1-1	Name (LAST, First)	JOIKE, Trevor B.
IV-1-2	Address	Schiff Hardin LLP 6600 Sears Tower Chicago, Illinois 60606 United States of America
IV-1-3	Telephone No.	312 258-5774
IV-1-4	Facsimile No.	312 258-5921
IV-1-5	e-mail	tjoike@schiffhardin.com
IV-1-6	Agent's registration No.	25,542
V	DESIGNATIONS	
V-1	The filing of this request constitutes under Rule 4.9(a), the designation of all Contracting States bound by the PCT on the international filing date, for the grant of every kind of protection available and, where applicable, for the grant of both regional and national patents.	
VI-1	Priority claim of earlier national application	
VI-1-1	Filing date	05 December 2003 (05.12.2003)
VI-1-2	Number	10/729,722
VI-1-3	Country	US
VI-2	Priority document request The receiving Office is requested to prepare and transmit to the International Bureau a certified copy of the earlier application(s) identified above as item(s):	VI-1
VII-1	International Searching Authority Chosen	European Patent Office (EPO) (ISA/EP)

PCT REQUEST

Original (for SUBMISSION)

VIII	Declarations	Number of declarations	
VIII-1	Declaration as to the identity of the inventor	-	
VIII-2	Declaration as to the applicant's entitlement, as at the international filing date, to apply for and be granted a patent	1	
VIII-3	Declaration as to the applicant's entitlement, as at the international filing date, to claim the priority of the earlier application	1	
VIII-4	Declaration of inventorship (only for the purposes of the designation of the United States of America)	-	
VIII-5	Declaration as to non-prejudicial disclosures or exceptions to lack of novelty	-	

PCT REQUEST

Original (for SUBMISSION)

VIII-2-1	Declaration: Entitlement to apply for and be granted a patent Declaration as to the applicant's entitlement, as at the international filing date, to apply for and be granted a patent (Rules 4.17(ii) and 51bis.1(a)(ii)). In a case where the declaration under Rule 4.17(iv) is not appropriate: Name (LAST, First)	in relation to this international application ZENITH ELECTRONICS CORPORATION is entitled to apply for and be granted a patent by virtue of the following:
VIII-2-1(i) v)		an assignment from KIM, Jin H. to ZENITH ELECTRONICS CORPORATION, dated 05 December 2003 (05.12.2003)
VIII-2-1(i) v)		an assignment from OZEN, Serdar to ZENITH ELECTRONICS CORPORATION, dated 04 December 2003 (04.12.2003)
VIII-2-1(i) x)	This declaration is made for the purpose of:	all designations except the designation of the United States of America

PCT REQUEST

Original (for SUBMISSION)

VIII-3-1	Declaration: Entitlement to claim priority Declaration as to the applicant's entitlement, as at the international filing date, to claim the priority of the earlier application specified below, where the applicant is not the applicant who filed the earlier application or where the applicant's name has changed since the filing of the earlier application (Rules 4.17(II) and 51bis.1(a)(II)) Name	in relation to this international application ZENITH ELECTRONICS CORPORATION is entitled to claim priority of earlier application No. 10/729,722 by virtue of the following:
VIII-3-1(i) v)		an assignment from KIM, Jin H. to ZENITH ELECTRONICS CORPORATION, dated 05 December 2003 (05.12.2003)
VIII-3-1(i) v)		an assignment from OZEN, Sardar to ZENITH ELECTRONICS CORPORATION, dated 04 December 2003 (04.12.2003)
VIII-3-1(i) x)	This declaration is made for the purposes of:	all designations

PCT REQUEST

Original (for SUBMISSION)

IX	Check list	number of sheets	electronic file(s) attached
IX-1	Request (including declaration sheets)	6	✓
IX-2	Description	36	-
IX-3	Claims	14	-
IX-4	Abstract	1	✓
IX-5	Drawings	2	-
IX-7	TOTAL	59	
	Accompanying items	paper-document(s) attached	electronic file(s) attached
IX-8	Fee calculation sheet	✓	-
IX-17	PCT-SAFE physical media	-	✓
IX-18	Figure of the drawings which should accompany the abstract	2	
IX-20	Language of filing of the international application	English	
X-1	Signature of applicant, agent or common representative		
X-1-1	Name (LAST, First)	JOIKE, Trevor S.	
X-1-2	Name of signatory		
X-1-3	Capacity		

FOR RECEIVING OFFICE USE ONLY

10-1	Date of actual receipt of the purported international application	
10-2	Drawings:	
10-2-1	Received	
10-2-2	Not received	
10-3	Corrected date of actual receipt due to later but timely received papers or drawings completing the purported international application	
10-4	Date of timely receipt of the required corrections under PCT Article 11(2)	
10-5	International Searching Authority	ISA/EP
10-6	Transmittal of search copy delayed until search fee is paid	

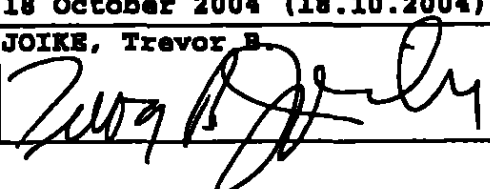
FOR INTERNATIONAL BUREAU USE ONLY

11-1	Date of receipt of the record copy by the International Bureau	
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PCT (ANNEX - FEE CALCULATION SHEET)

Original (for SUBMISSION)

(This sheet is not part of and does not count as a sheet of the international application)

0	For receiving Office use only	
0-1	International Application No.	
0-2	Date stamp of the receiving Office	
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...forming a channel estimate, a received
signal y is decoded to form data s , a convolution matrix
 S is formed from the data s , a matrix F is formed from
the data $\hat{s}$ such that the matrix F results from the
forming of the matrix S as a convolution matrix, and a
conjugate gradient algorithm is performed to determine
the channel estimate. The conjugate gradient algorithm
is based on the received signal y , the matrix S , and the
matrix F .

EFFICIENT CONJUGATE GRADIENT
BASED CHANNEL ESTIMATOR

Technical Field of the Invention

5 The present invention relates to a conjugate
gradient based channel estimator.

Background of the Invention

 Since the adoption of the ATSC digital
10 television (DTV) standard in 1996 in the United States,
there has been an ongoing effort to improve the design of
receivers built for the ATSC DTV signal. The primary
obstacle that faces designers in designing receivers so
that they achieve good reception is the presence of
15 multipath interference in the channel.

 The broadcast television channel is a
relatively severe multipath environment due to a variety
of conditions that are encountered in the channel and at
the receiver. Channels are characterized by impulse
20 responses which may be several hundreds of symbols long,
so that strong interfering signals may arrive at the
receiver both before and after the largest amplitude
signal. In addition, the signal transmitted through the
channel is subject to time varying channel conditions due
25 to the movement of the transmitter and signal reflectors,
airplane flutter, and, for indoor reception, people

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walking around the room. If mobile reception is desired, movement of the receiver must also be considered.

Moreover, the ATSC DTV signal uses trellis coded 8-level vestigial sideband (usually referred to as 8T-VSB or, more simply, as 8-VSB) as the modulation method. 8-VSB data symbols are real and have a signal pulse shape that is complex. Only the real part of the complex pulse shape is a Nyquist pulse. Therefore, even if there is no multipath, the imaginary part of the complex pulse shape contributes intersymbol interference (ISI) when the channel gain seen by the equalizer is not real.

Multipath and intersymbol interference adversely affects the ability of the receiver to correctly receive the symbols transmitted by the transmitter. Therefore, designers add equalizers to receivers in order to cancel the effects of multipath and intersymbol interference and thereby improve signal reception.

Because the channel is not known *a priori* at the receiver, the equalizer must be able to modify its response to match the channel conditions that it encounters and to adapt to changes in those channel conditions. To aid in the convergence of an adaptive

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equalizer to channel conditions, the field sync segment of the frame as defined in the ATSC standard may be used as a training sequence for the equalizer. But when equalization is done in the time domain, long equalizers
5 (those having many taps) are required due to the long channel impulse responses that characterize the channel.

The original Grand Alliance receiver used an adaptive decision feedback equalizer (DFE) with 256 taps. This adaptive decision feedback equalizer was adapted to
10 the channel using a standard least mean square (LMS) algorithm, and was trained with the field sync segment of the transmitted frame. The LMS algorithm converges quite slowly and, even with only 256 taps, does not always converge during a single training sequence. Because the
15 field sync segment is transmitted relatively infrequently (about every 260,000 symbols), the total convergence time of this equalizer is quite long if the equalizer only adapts on training symbols prior to convergence.

Therefore, in order to adapt equalizers to
20 follow channel variations that occur between training sequences, the addition of blind and decision directed methods to equalizers has been suggested. However, when implemented in a realistic system, these methods may require several data fields to achieve convergence, and

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convergence may not be achieved at all under difficult multipath conditions.

In any event, because multipath signals in the broadcast channel may arrive many symbols after the main
5 signal, the decision feedback equalizer is invariably used in 8-VSB applications.

It has been argued that a blind decision feedback equalizer is required due to the rise in mean square error (MSE) between training sequences. However,
10 adaptation of the trained equalizer in the simulation that supported this argument was frozen between training sequences. It is possible that a decision-directed equalizer with good tracking performance may be able to follow the channel variations tested.

15 Blind algorithms based on the Sato algorithm and on Godard's constant modulus algorithm (CMA) have been proposed. The error term in both of these algorithms uses a continuous blending of a decision-directed term with the blind term. This blending enables
20 a smooth transition between the blind mode and the decision-directed mode. However, when implemented in a realistic system, these algorithms also may take several data fields to converge.

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As mentioned previously, adaptive equalizers utilizing the least mean square (LMS) algorithm may converge slowly or not at all depending on the channel conditions. Convergence may be adversely affected if the input data auto-correlation matrix has a large eigenvalue spread. Also, if the decision feedback equalizer has not converged before the end of the training sequence, the shape of the objective function may change so that it includes local minima. These local minima may be caused by closed eye channel conditions and decision feedback equalizer error propagation.

The recursive least square (RLS) algorithm is well known to avoid these convergence problems. However, the recursive least square algorithm, in its basic form, requires the computationally intensive inversion of the input data auto-correlation matrix. Lattice based forms of the recursive least square algorithm avoid the need for this matrix inversion. However, the lattice based forms of the recursive least square algorithm are not easily amenable to the advantage of initialization from an initial channel impulse response (CIR) estimate, even though such an initialization may be desirable.

Recent work in reduced rank filtering has connected the multi-stage nested Wiener filter (MSNWF) of

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Goldstein and Reed to the conjugate gradient algorithm (CG). It has been shown that the multi-stage nested Wiener filter solves the Wiener-Hopf equations in the Krylov subspace associated with the auto-correlation matrix and the cross-correlation vector. The multi-stage nested Wiener filter is then re-formulated using the Lanczos iteration. It has also been shown that the Lanczos-based multi-stage nested Wiener filter is equivalent to the conjugate gradient algorithm. Since the multi-stage nested Wiener filter often needs few dimensions to approach the performance of the full-rank Wiener filter, the conjugate gradient algorithm is a good candidate for an adaptive equalization algorithm with fast convergence.

15 A description of the conjugate gradient optimization algorithm and many of its mathematical properties may be found in "An Introduction to Optimization," by E. K. P. Chong and S. Zak, New York, NY, John Wiley & Sons, 1996. The algorithm described is applicable to the optimization of a fixed objective function. An excellent review of adaptive filtering with the conjugate gradient algorithm is found in "Analysis of conjugate gradient algorithms for adaptive filtering," by P. S. Chang and A. N. Willson, Jr., vol. 48, pp. 409-418,

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IEEE Transactions on Signal Processing, February 2000.

The mathematical richness of the algorithm relationships leads to a variety of options in the implementation of the basic conjugate gradient algorithm for minimizing a quadratic objective function. These options can be carried over to the adaptive situation to provide a number of options for implementation of the algorithm. One characteristic of the algorithm which may also be exploited is that it works directly with the correlation matrices and vectors. This characteristic makes initialization based on an initial channel impulse response straightforward, assuming information is available for this purpose.

Like lattice based forms of the recursive least square algorithm, the conjugate gradient algorithm does not require the inversion of the input data auto-correlation matrix. Another characteristic of the conjugate gradient algorithm may be exploited by recognizing that the input data auto-correlation matrix is the product of two Toeplitz matrices.

The invention described in U.S. Patent Application Serial No. (7227) uses these characteristics to substantially reduce the computation load in the conjugate gradient algorithm and to even eliminate the

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need to form the input auto-correlation matrix.

Accordingly, the improved conjugate gradient algorithm disclosed in that application may be used to achieve satisfactory convergence times for equalizers.

5 The (7227) application also suggests that the conjugate gradient algorithm can be used to estimate channels. The present invention reduces the computation complexity in performing the conjugate gradient algorithm with respect to channel estimators.

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Summary of the Invention

In accordance with one aspect of the present invention, a method of processing a received signal y to produce a channel estimate comprises the following: (a)
15 decoding the received signal y to form data s ; (b) forming a convolution matrix $\hat{S}$ from the data s ; (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and, (d) performing a conjugate gradient
20 algorithm to determine the channel estimate, wherein the conjugate gradient algorithm is based on the received signal y , the matrix $\hat{S}$, and the matrix F .

In accordance with another aspect of the present invention, a method of processing a received

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signal y comprises the following: (a) decoding the received signal y to form data s ; (b) forming a convolution matrix $\hat{S}$ from the data s ; (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and, (d) performing a conjugate gradient algorithm based on the received signal y , the matrix $\hat{S}$, and the matrix F .

In accordance with still another aspect of the present invention, a method of processing a received signal y comprises the following: (a) decoding the received signal y to form data s ; (b) forming a convolution matrix $\hat{S}$ from the data s ; (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and, (d) performing a conjugate gradient algorithm based on the received signal y , the matrix $\hat{S}$, and the matrix F , wherein the conjugate gradient algorithm includes forming FFTs based on the received signal y , the matrix $\hat{S}$, and the matrix F , multiplying the FFTs to form a multiplication product, and forming an inverse FFT of the multiplication product.

In accordance with yet another aspect of the present invention, a method of processing a received signal y to produce a channel estimate comprises the

following: (a) decoding the received signal y to form data s ; (b) forming a convolution matrix $\hat{S}$ from the data s ; (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and, (d) performing a conjugate gradient algorithm to determine the channel estimate, wherein the conjugate gradient algorithm is based on the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein the conjugate gradient algorithm includes forming FFTs based on the received signal y , the matrix $\hat{S}$, and the matrix F , multiplying the FFTs to form a multiplication product, and forming an inverse FFT of the multiplication product.

15 Brief Description of the Drawings

These and other features and advantages will become more apparent from a detailed consideration of the invention when taken in conjunction with the drawings in which:

20 Figure 1 illustrates a filter that can be used in a decision feedback equalizer according to an embodiment of the present invention;

Figure 2 illustrates the decision feedback equalizer according to an embodiment of the present invention;

Figure 3 illustrates a matrix S which is
5 derived from a vector of symbol decisions of length 2496 and is useful in describing the present invention;

Figure 4 illustrates the decomposition of the matrix S to form two matrices $\hat{S}$ and F useful in implementing the present invention; and,

10 Figure 5 shows a system for determining a channel estimate h based on the matrices $\hat{S}$ and F .

Detailed Description

As shown in Figure 1, a filter 10 can be used
15 as an equalizer and includes storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$. For example, N may be 512. The storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$ store corresponding received data $y[n]$, ..., $y[n+N-3]$, $y[n+N-2]$, $y[n+N-1]$. The output of
20 each of the storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$ is coupled to a corresponding multiplier $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$. Accordingly, the multipliers $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$ receive the stored data from the

storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$,
 $12[n+N-1]$. The multipliers $14[n]$, ..., $14[n+N-3]$,
 $14[n+N-2]$, $14[n+N-1]$ also receive corresponding tap
weights $g[n]$, ..., $g[n+N-3]$, $g[n+N-2]$, $g[n+N-1]$. The
5 multipliers $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$
multiply each of the stored data $y[n]$, ..., $y[n+N-3]$,
 $y[n+N-2]$, $y[n+N-1]$ by a corresponding one of the tap
weights $g[n]$, ..., $g[n+N-3]$, $g[n+N-2]$, $g[n+N-1]$, and the
multiplication results are summed by a summer 16 to
10 provide the output of the filter 10. Various algorithms
as described above have been implemented in order to set
the values of the tap weights $g[n]$, ..., $g[n+N-3]$, $g[n+N-2]$, $g[n+N-1]$.

A decision feedback equalizer 20 is shown in
15 Figure 2 and includes a feed forward filter 22, a
feedback filter 24, a summer 26, a hard decision device
28, and a controller 30 that computes the tap weights for
the feed forward filter 22 and the feedback filter 24.
Each of the feed forward filter 22 and the feedback
20 filter 24 may comprise the filter 10 of Figure 1. The
hard decision device 28, for example, may be a decoder or
a slicer.

The output $\hat{s}[n]$ of the summer 26 is the
equalized output of the decision feedback equalizer 20.

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The hard decision device 28 (such as a slicer) determines the closest value $\hat{s}[n]$ for each of the symbols provided as the output $\hat{s}[n]$ of the summer 26. These values $\hat{s}[n]$ are fed back as inputs to the feedback filter 24. The
5 feedback filter 24 applies tap weights $\hat{g}_s[n]$ to the values $\hat{s}[n]$ and provides its output to the summer 26. Likewise, the feed forward filter 22 applies tap weights $\hat{g}_f[n]$ to the values $y[n]$ of the received signal and provides its output to the summer 26. The summer 26 sums the output
10 of the feedback filter 24 with the output of the feed forward filter 22 in order to produce the output $\hat{s}[n]$.

The output $\hat{s}[n]$ of the decision feedback equalizer 20 is given by the following equation:

$$\begin{aligned} \hat{s}[n] &= \text{Re}\{g_f^H y[n]\} + g_s^T \hat{s}[n] \\ &= g_{fR}^T y_R[n] + g_{fI}^T y_I[n] + g_s^T \hat{s}[n] \end{aligned} \quad (1)$$

where $\hat{s}[n]$ as indicated above is defined to be the symbol estimates provided by the hard decision device 28, where
20 $g_{fR} = \text{Re}\{g_f\}$ is the real part of the tap weights g_f of the feed forward filter 22, where $g_{fI} = \text{Im}\{g_f\}$ is the imaginary part of the tap weights g_f of the feed forward filter 22,

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and where g_s are the tap weights of the feedback filter
24.

The definitions given by the following
equations may be assumed:

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$$\tilde{\mathbf{y}}_r[n] = [\mathbf{y}_r^T[n] \mathbf{y}_i^T[n]]^T \quad (2)$$

$$\tilde{\mathbf{y}}[n] = [\tilde{\mathbf{y}}_r^T[n] \tilde{\mathbf{s}}^T[n]]^T \quad (3)$$

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$$\tilde{\mathbf{g}}_r[n] = [\mathbf{g}_m^T[n] \mathbf{g}_{rr}^T[n]]^T \quad (4)$$

$$\tilde{\mathbf{g}}[n] = [\tilde{\mathbf{g}}_r^T[n] \tilde{\mathbf{g}}_s^T[n]]^T \quad (5)$$

where $\mathbf{y}_r = \text{Re}\{\mathbf{y}_r\}$ is the real part of the received symbols
15 $\mathbf{y}_r$, and where $\mathbf{y}_i = \text{Im}\{\mathbf{y}_r\}$ is the imaginary part of the
received symbols $\mathbf{y}_r$. Then, the equalized symbols $\hat{\mathbf{s}}[n]$
provided by the decision feedback equalizer 20 are given
by the following equation:

20

$$\begin{aligned} \hat{\mathbf{s}}[n] &= \tilde{\mathbf{g}}_r^T \tilde{\mathbf{y}}_r[n] + \mathbf{g}_s^T \hat{\mathbf{s}}[n] \\ &= \tilde{\mathbf{g}}^T \tilde{\mathbf{y}}[n] \end{aligned} \quad (6)$$

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The equalizer input $y[n]$ and the feed forward tap weights g_f are complex values, the feedback tap weights g_b are real, and the output $\hat{s}[n]$ of the decision feedback equalizer 20 is real. The imaginary part of the output of the feed forward filter 22, g_{ff} , may be ignored and, therefore, not computed.

It is desired to determine the tap weight vector $\tilde{g}[n]$ so that the error at the output of the decision feedback equalizer 20 is minimized according to the following equations:

$$\tilde{g}[n] = \arg \min_g \frac{1}{2} E\{s[n] - \hat{s}[n]\}^2 \quad (7)$$

$$\begin{aligned} F(g) &= E\{s[n] - \hat{s}[n]\}^2 \\ &= \sigma_s^2 - \tilde{g}^H[n] x_{sy} - x_{sy}^H \tilde{g}[n] + \tilde{g}^H[n] R_{yy} \tilde{g}[n] \end{aligned} \quad (8)$$

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where σ_s^2 is the symbol variance and is equal to the average symbol energy, R_{yy} is an autocorrelation matrix, and x_{sy} is a cross correlation vector. The autocorrelation matrix R_{yy} and the cross correlation vector x_{sy} are defined in detail hereinafter. The error function $F(g)$ is minimized by setting its gradient to zero according to the following equation:

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$$\nabla F(g) = R_{yy}\tilde{g}[n] - r_{sy} = 0 \quad (9)$$

This minimization leads to the following well known

5 Weiner-Hopf equation:

$$R_{yy}\tilde{g}[n] = r_{sy} \quad (10)$$

Solving for the desired tap weights $\tilde{g}[n]$ leads to the

10 following equation:

$$\tilde{g}[n] = R_{yy}^{-1}r_{sy} \quad (11)$$

So, in order to calculate the desired tap weight vector,
15 the large system of equations represented by equation
(11) must be solved. This solution requires the
inversion of the large matrix R_{yy} and, therefore, a
considerable amount of computation.

One method for reducing the amount of
20 computation involved in solving a large system of
equations is the aforementioned conjugate gradient
algorithm. In order to set up equation (11), estimation
of the auto-correlation matrix $R_{yy}[n=0]$ and the cross
correlation vector $r_{sy}[n=0]$ is required. By definition,

the auto-correlation matrix $R_{yy}[n]$ is given by the following equation:

$$R_{yy}[n] = E[\tilde{y}[n]\tilde{y}^H[n]] \quad (12)$$

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and the cross correlation vector $r_{sy}[n]$ is given by the following equation:

$$r_{sy}[n] = E[s[n]\tilde{y}[n]] \quad (13)$$

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where $s[n]$ represents the transmitted symbols, E is the expectation operator, and H denotes the Hermitian transpose.

The auto-correlation matrix $R_{yy}[n]$ and the cross correlation vector $r_{sy}[n]$ can be estimated using the forgetting factor method according to the following equations:

$$\begin{aligned} \hat{R}_{yy}[n] &= \sum_{m=0}^{n-1} \gamma^m \tilde{y}[n-m]\tilde{y}^H[n-m] \\ &= \gamma \hat{R}_{yy}[n-1] + \tilde{y}[n]\tilde{y}^H[n] \end{aligned} \quad (14)$$

20

$$\hat{r}_{sy}[n] = \sum_{m=0}^{n-1} \gamma^m s[n-m]\tilde{y}^H[n-m]$$

$$= \gamma \hat{\mathbf{x}}_{\mathbf{y}}[n-1] + \mathbf{s}[n] \tilde{\mathbf{y}}^H[n] \quad (15)$$

where $0 < \gamma < 1$ is the forgetting factor. If γ is set to 0, the equations (14) and (15) yield the estimates of the auto-correlation matrix $\hat{\mathbf{R}}_{\mathbf{y}\mathbf{y}}$ and the cross correlation vector $\hat{\mathbf{x}}_{\mathbf{y}}$ from the most recent sample vector $\mathbf{y}$.

In determining the tap weights $\mathbf{g}_f$ for the feed forward filter 22 and $\mathbf{g}_b$ for the feedback filter 24, the controller 30 initializes the auto-correlation matrix $\mathbf{R}_{\mathbf{y}\mathbf{y}}[0]$ using equation (14) and the cross correlation vector $\mathbf{x}_{\mathbf{y}}[0]$ using equation (15) by performing averaging over a period of time such as 1/2 data field or so. The controller 30 also initializes the tap weights $\tilde{\mathbf{g}}_0[0] = 0$ and initializes the decision vector $\mathbf{s}[0]$ to the first N_{tb} of the known training symbols that are contained in the field sync portion of each data field as discussed above.

Further, the controller 30 initializes the data vector $\tilde{\mathbf{y}}[0]$ as follows. The data vector $\tilde{\mathbf{y}}_r[0]$ as given in equation (2) is initialized by setting the real part $\mathbf{y}_R[n=0] = \{\mathbf{y}_R[0], \dots, \mathbf{y}_R[N_{eff}-1]\}^T$ such that $\mathbf{y}_R[0], \dots, \mathbf{y}_R[N_{ts}-N_{tb}-1]$ are the real part of the remaining training data (after those in $\tilde{\mathbf{s}}[0]$) and

$y_R[N_{ts} - N_{fb}], \dots, y_R[N_{ff} - 1]$ are the real part of the received data after the training sequence. This setting is repeated for $\tilde{y}_r[0]$. Then, $\tilde{y}_r[0] = [y_R^T[0] y_I^T[0]]^T$ is constructed as given in equation (2) and $\tilde{y}[0] = [\tilde{y}_r^T[0] \tilde{a}^T[0]]^T$ is constructed as given in equation (3) based on the initialized vectors $\tilde{y}_r[0]$, $\tilde{y}_i[0]$, and $\tilde{a}[0]$ as given above. In an ATSC digital television environment, for example, the number of training symbols N_{ts} may be 728, the number of feed forward taps N_{ff} may be 512, and the number of feedback taps N_{fb} may also be 512.

Further, the controller 30 initializes the output $\hat{s}[n]$ of the decision feedback equalizer 20 as $\hat{s}[0] = \hat{g}_0^T[0] \tilde{y}[0]$ in accordance with equation (6), the controller 30 initializes a gradient $t_n[n]$ as $t_0[0] = \hat{R}_{yy}[0] \tilde{g}_0[0] - \hat{e}_{sy}[0]$, and the controller 30 sets the variable n to 0.

The controller 30 then executes a conjugate gradient algorithm once for each received data element, where each iteration of the algorithm is initiated upon the shifting of a new received data element into the feed forward filter 22. The parameters of the conjugate gradient algorithm as described above are defined in the table.

Table

Parameter Name	Equalizer Notation
Object Function Argument (Equalizer)	$\mathbf{g}_k[n]$
Argument Step Size	$\alpha_k[n]$
Gradient	$\mathbf{t}_k[n]$
Conjugate Directions	$\mathbf{b}_k[n]$
Conjugate Direction Step Size	$\beta_k[n]$

5 This conjugate gradient algorithm is as follows:

1. calculate conjugate direction $\mathbf{b}_k[n]$

if $n \bmod N_{\text{taps}} = 0$

$\mathbf{b}_0[n] = -\mathbf{t}_0[0]$ (reset)

else

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$$\beta_0[n] = \frac{(\mathbf{t}_0[n] - \mathbf{t}_{-1}[n])^H \mathbf{t}_0[n]}{(\mathbf{t}_0[n] - \mathbf{t}_{-1}[n])^H \mathbf{b}_{-1}[n]}$$

$$\mathbf{b}_0[n] = -\mathbf{t}_0[n] + \beta_0[n] \mathbf{b}_{-1}[n]$$

2. calculate argument step size $\alpha_k[n]$

$$\alpha_0[n] = \frac{-\mathbf{b}_0^H[n] \mathbf{t}_0[n]}{\mathbf{b}_0^H[n] \hat{\mathbf{R}}_{yy}[n] \mathbf{b}_0[n]}$$

3. tap update

$$\tilde{\mathbf{g}}_1[n] = \tilde{\mathbf{g}}_0[n] + \alpha_0[n]\mathbf{b}_0[n]$$

4. $n = n + 1$

$$\tilde{\mathbf{g}}_0[n] = \tilde{\mathbf{g}}_1[n - 1]$$

$$\mathbf{t}_{-1}[n] = \mathbf{t}_0[n - 1]$$

5 $\mathbf{b}_{-1}[n] = \mathbf{b}_0[n - 1]$

5. update input vector $\tilde{\mathbf{y}}[n]$ with the next received data element and update $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}[n]$ per equation (14)

6. Calculate output of the decision feedback equalizer 20

$$\hat{\mathbf{s}}[n] = \tilde{\mathbf{g}}_0^T[n]\tilde{\mathbf{y}}[n]$$

7. calculate gradient $\mathbf{t}_k[n]$

$$\mathbf{t}_0[n] = \mathbf{t}_{-1}[n] + \alpha_0[n][\hat{\mathbf{R}}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}[n]\mathbf{b}_0[n]]$$

8. Return to step 1

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The largest of the part of the computations required by this conjugate gradient algorithm is contributed by the matrix-vector multiplications $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}[n]\mathbf{b}_0[n]$ of steps 2 and 7. These multiplications require the update of matrix $\hat{\mathbf{R}}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}[n]$ from the vector $\tilde{\mathbf{y}}[n]$ as in equation (14). This update requires one multiply for each of the N_{taps}^2 matrix elements, a total of N_{taps}^2 multiplies. Then, the matrix-vector multiplication

requires an additional N_{taps}^2 multiplies. Accordingly, the total computation is $N_{\text{taps}}^2 + N_{\text{taps}}^2 = 2N_{\text{taps}}^2$ multiplies.

This computational load can be decreased as explained below.

5 A matrix-vector multiplication given by $\mathbf{a} = \mathbf{Y}\mathbf{b}$, where $\mathbf{Y}$ is a convolution matrix, $\mathbf{b}$ is a vector, and $\mathbf{a}$ is the multiplication result, is equivalent to $a[n] = y[n] * b[n]$ where $*$ denotes linear convolution. These equations may be represented in matrix form by the
10 following equation:

$$\begin{bmatrix} a[0] \\ a[1] \\ a[2] \end{bmatrix} = \begin{bmatrix} y[0] & y[-1] & y[-2] \\ y[1] & y[0] & y[-1] \\ y[2] & y[1] & y[0] \end{bmatrix} \begin{bmatrix} b[0] \\ b[1] \\ b[2] \end{bmatrix} \quad (16)$$

where n is chosen as a small number for explanatory
15 purposes only. The columns and rows of the matrix $\mathbf{Y}$ in equation (16) can be circularly extended in accordance with the following equation:

$$\begin{bmatrix} a[0] \\ a[1] \\ a[2] \\ dc \\ dc \end{bmatrix} = \begin{bmatrix} y[0] & y[-1] & y[-2] & y[2] & y[1] \\ y[1] & y[0] & y[-1] & y[-2] & y[2] \\ y[2] & y[1] & y[0] & y[-1] & y[-2] \\ y[-2] & y[2] & y[1] & y[0] & y[-1] \\ y[-1] & y[-2] & y[2] & y[1] & y[0] \end{bmatrix} \begin{bmatrix} b[0] \\ b[1] \\ b[2] \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

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where dc means don't care. Equation (17) is equivalent to $a[n] = y[n] \otimes b[n]$ where $\otimes$ denotes circular convolution. Circular convolution is equivalent to the following operation:

5

$$\mathcal{Y} = \text{FFT}(\mathbf{y}) \quad (18)$$

$$\mathcal{B} = \text{FFT}(\mathbf{b}) \quad (19)$$

$$\mathcal{A} = \mathcal{Y} \times \mathcal{B} \quad (20)$$

$$\mathbf{a} = \text{IFFT}(\mathcal{A}) \quad (21)$$

10

where FFT in equations (18) and (19) indicates the Fast Fourier Transform operation, IFFT in equation (21) indicates the Inverse Fast Fourier Transform operation, and $\times$ in equation (20) indicates point-wise vector

15 multiplication such that the elements of the resultant vector are given by $\mathbf{Y}_0 \times \mathbf{B}_0$, $\mathbf{Y}_1 \times \mathbf{B}_1$, $\mathbf{Y}_2 \times \mathbf{B}_2$, etc.

An embellishment on the above process involves 0 padding the columns to lengths of 2^n . This 0 padding, as is well known, permits a computationally efficient
20 FFT. Accordingly, the matrix-vector multiplication $\mathbf{a} = \mathbf{Yb}$ can be written with 0 padding in matrix form according to the following equation:

25

$$\begin{bmatrix} a[0] \\ a[1] \\ a[2] \\ dc \\ dc \\ dc \\ dc \\ dc \end{bmatrix} = \begin{bmatrix} y[0] & y[-1] & y[-2] & 0 & 0 & 0 & y[2] & y[1] \\ y[1] & y[0] & y[-1] & y[-2] & 0 & 0 & 0 & y[2] \\ y[2] & y[1] & y[0] & y[-1] & y[-2] & 0 & 0 & 0 \\ 0 & y[2] & y[1] & y[0] & y[-1] & y[-2] & 0 & 0 \\ 0 & 0 & y[2] & y[1] & y[0] & y[-1] & y[-2] & 0 \\ 0 & 0 & 0 & y[2] & y[1] & y[0] & y[-1] & y[-2] \\ y[-2] & 0 & 0 & 0 & y[2] & y[1] & y[0] & y[-1] \\ y[-1] & y[-2] & 0 & 0 & 0 & y[2] & y[1] & y[0] \end{bmatrix} \begin{bmatrix} b[0] \\ b[1] \\ b[2] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (22)$$

It is noted that matrix Y has a Toeplitz structure, and that Y^H is also a Toeplitz matrix. (H denotes Hermitian transpose.)

It will be shown that the need for both the R_{yy} update and the $R_{yy}b$ multiply can be eliminated, and that the matrix R_{yy} need not even be created.

Let $\gamma = 0$ for equation (14). It is well known that

$$R_{yy} = \tilde{y}\tilde{y}^H = YY^H$$

where Y is a convolution matrix based on the vector y (that is, y forms the first column of the matrix Y). The vector $\tilde{y}$ may be 0 padded to length 2^n . Let $\tilde{y}'$ be the first column of Y^H . For the conjugate gradient operation, the calculation given by the following equation must be made:

$$v = \hat{R}_{yy}[n]b_0[n] = Y^H Y b_0[n] \quad (23)$$

In order to calculate v , the quantity $Y^H b_0[n]$ should first
5 be calculated according to the following equations:

$$Y = \text{FFT}(y) \quad (24)$$

$$B = \text{FFT}(b) \quad (25)$$

$$X = Y \times B \quad (26)$$

$$10 \quad x = \text{IDFT}(X) \quad (27)$$

$$x_N = \text{first } N \text{ elements of } x, \text{ remaining elements} = 0 \quad (28)$$

$$X_N = \text{FFT}(x_N) \quad (29)$$

where x in equation (26) denotes point-wise vector
15 multiplication. Then, $Y \times X_N$ is calculated according to
the following equations:

$$Y' = \text{FFT}(Y') \quad (30)$$

$$V = Y' \times X \quad (31)$$

$$20 \quad v = \text{IDFT}(V) \quad (32)$$

$$v_N = \text{first } N \text{ elements of } v, \text{ remaining elements} = 0 \quad (33)$$

The number of multiplies in this process, particularly for large N_{taps} , is much less than $2N_{\text{taps}}^2$.

In the process given by equations (24)-(33), $\mathbf{b}$ is the vector $\mathbf{b}_0$ of the eight step algorithm described above, $\mathbf{y}$ is a vector of received symbols as described above, and $\mathbf{v}_N$ is the quantity $\hat{\mathbf{R}}_{yy}[n]\mathbf{b}_0[n]$ in steps 2 and 7 of the conjugate gradient algorithm described above.

The conjugate gradient algorithm described above can be used to make other calculations. For example, the conjugate gradient algorithm described above can be used to calculate updated channel impulse response estimates. This calculation can be useful, for instance, because known alternative methods of calculating MMSE (Minimum Mean Square Error) equalizer tap weights are based on having a channel impulse response estimate.

The conjugate gradient method described above permits the continuous calculation of channel impulse response estimate updates between training sequences (when only a priori unknown symbols are received) in a receiver for a time varying multipath channel. This calculation utilizes a least squares channel impulse response estimation calculated from an over-determined system of equations. Dynamic changes to the channel impulse response may be tracked by using receiver trellis

decoder decisions on input symbols to form a long sequence of almost perfectly known symbols. This sequence should have relatively few errors.

The channel impulse response to be estimated is of length $L_h = L_{ha} + L_{hc} + 1$ where L_{ha} is the length of the anti-causal part of the channel impulse response and L_{hc} is the length of the causal part of the channel impulse response.

The reliable trellis decoder decisions on the input symbols is a vector designated herein as $\mathbf{s}$ of length L_s . A Toeplitz matrix S may be defined based on this vector $\mathbf{s}$ according to the following equation:

$$\mathbf{S} = \begin{bmatrix} s_{L_h-1} & s_{L_h-2} & - & - & - & - & - & s_0 \\ - & s_{L_h-1} & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & s_{L_h-1} & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & s_{L_s-L_h} & - & - \\ s_{L_s-1} & s_{L_s-2} & - & - & - & - & - & s_{L_s-L_h} \end{bmatrix} \quad (34)$$

15

where the elements in the matrix of equation (34) are real and consist of the symbol decisions of vector $\mathbf{s}$.

To ensure an over-determined system of equations, the following inequality is necessary:

20

$$L_s \geq 2L_h - 1. \quad (35)$$

The matrix S is of dimension $(L_s - L_h + 1) \times L_h$ with
 $(L_s - L_h + 1) \geq L_h$. The received signal vector is y with
 5 data elements y_i for $L_{hc} \leq i \leq (L_s - L_{ha} - 1)$ where y_i is
 the received symbol corresponding to input symbol
 decision s_i . Then, $y = Sh + v$ where h is the L_h long
 channel impulse response vector and v is a white noise
 vector.

10 The least squares solution for h is given by
 the following equation:

$$\hat{h}_{LS} = (S^T S)^{-1} S^T y. \quad (36)$$

15 By utilizing reliable trellis decoder input
 symbol decisions, there should be sufficient support for
 calculating a channel impulse response estimate of the
 required length. As required by inequality (35), the
 vector s of symbol decisions must be at least twice as
 20 long as the channel impulse response being estimated.

For an 8 VSB receiver application, the
 following parameters may be assumed: $L_h = 512$, $L_{ha} = 63$,
 $L_{hc} = 448$, and $L_s = 2496$. The vector s may be formed from
 a sequence of trellis decoder decisions on the input

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symbols. Normally, the trellis decoder would just make output bit pair decisions, but it can also make equally reliable decisions on the input symbols.

The vector $\mathbf{a}$, for example, may be selected as 3
5 segments ($L_s = 2496$ symbols) long. So, three data segments may be used to produce a single channel impulse response estimate update. A new channel impulse response estimate update can be obtained once per segment by proceeding in a sliding window manner. Optionally,
10 several consecutive channel impulse response estimate updates can be averaged in order to further improve channel impulse response estimate accuracy if necessary. Of course, this additional step of averaging can be a problem if the channel impulse response is varying
15 rapidly.

A vector $\mathbf{b}$ with fewer than 3 segments of symbol decisions may be used, but as stated in inequality (35), the length of vector $\mathbf{b}$ must be at least twice as long as the channel impulse response to be estimated. Longer $\mathbf{b}$
20 vectors help to diminish the adverse effect of additive white Gaussian noise on the channel.

The system of equations represented by equation (36) may be solved using the conjugate gradient algorithm in a manner similar to that previously described to solve

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equation (11) for the MMSE tap weights where $\mathbf{s}^T \mathbf{y}$ in
equation (36) takes the place of $\mathbf{x}_s^T \mathbf{y}$ in equation (11),
where $\mathbf{s}^T \mathbf{s}$ in equation (36) takes the place of $\mathbf{R}_{yy}$ in
equation (11), and where $\hat{\mathbf{h}}_{LS}$ in equation (36) takes the
5 place of $\tilde{\mathbf{g}}$ in equation (11).

So, the conjugate gradient algorithm described
above can facilitate the computation of $(\mathbf{s}^T \mathbf{s}) \mathbf{b}$ for the
case of the channel impulse response estimation just as
it facilitated the computation of $\mathbf{R}_{yy} \mathbf{b}$ for the
10 calculation of tap weights.

In further reducing the computation complexity
necessary to implement the matrix-vector multiplications
required to perform the conjugate gradient algorithm,
such as when the conjugate gradient algorithm is used to
15 estimate a channel as described above, it is useful to
note that the conjugate gradient algorithm may be used to
solve the following linear system for the channel
estimate $\mathbf{h}$:

$$20 \quad \mathbf{S} \mathbf{h} = \mathbf{y}. \quad (37)$$

The matrix $\mathbf{S}$ is an $m \times n$ (e.g., 1985 x 512) Toeplitz
matrix derived from symbol decisions, and $\mathbf{y}$ is a vector
(the observation vector) derived from the received data.

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The symbol decisions can be made, for example, by a trellis decoder or a slicer.

The system may be written as $\mathbf{S}^T \mathbf{S} \mathbf{h} = \mathbf{S}^T \mathbf{y}$ by multiplying both sides of equation (37) by $\mathbf{S}^T$. The
5 conjugate gradient algorithm can be used to solve this system for the channel estimate $\mathbf{h}$ according to the following steps:

(1) Calculate the residual vector $\mathbf{r}_1$ representing the
10 system $\mathbf{S}^T \mathbf{y} - \mathbf{S}^T \mathbf{S} \mathbf{h}_1$ as follows:

$$\mathbf{r}_1 = \mathbf{S}^T \mathbf{y} - \mathbf{S}^T \mathbf{S} \mathbf{h}_1 \quad (38)$$

where $\mathbf{r}_1$ is a 512 x 1 vector, and where $\mathbf{h}_1$ is the previous
15 channel estimate which may be initialized to any desired value such as $\mathbf{h}_1 = 0$.

(2) For $k = 1$ to n , iteratively calculate the following:

$$(a) \quad \mathbf{d}_k = \mathbf{r}_k + \beta_k \mathbf{d}_{k-1} \quad (39)$$

20

where $\beta_1 = 0$, $\beta_{k \geq 2} = \frac{\mathbf{r}_k^T \cdot \mathbf{r}_k}{\mathbf{r}_{k-1}^T \cdot \mathbf{r}_{k-1}}$, and where $\mathbf{d}_k$ is a 512 x 1

vector;

$$(b) \quad \mathbf{h}_{k+1} = \mathbf{h}_k + \alpha_k \mathbf{d}_k \quad (40)$$

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where $\mathbf{h}_{k+1}$ is a 512 x 1 vector, where α_k is 1 x 1, where

$$\alpha_k = \frac{\mathbf{r}_k^T \cdot \mathbf{r}_k}{\mathbf{d}_k^T \cdot \mathbf{q}_k}, \text{ where } \mathbf{q}_k = \mathbf{S}^T \mathbf{S} \mathbf{d}_k, \text{ where } \mathbf{S}^T \text{ is a } 512 \times 1985$$

matrix, and where $\mathbf{S}$ is a 1985 x 512 matrix; and,

5

$$(c) \quad \mathbf{r}_{k+1} = \mathbf{r}_k - \alpha_k \mathbf{q}_k \quad (41)$$

where $\mathbf{r}_{k+1}$ is a 512 x 1 vector.

Steps 2(a) - (c) are repeated enough times
10 until a desired accuracy is achieved for the channel
estimate $\mathbf{h}$. The index k , for example, may be
incremented up to a value of 5 although any other desired
value can be used. It will be seen that step 2(b)
includes the matrix vector multiplication $\mathbf{A}^T \mathbf{A} \mathbf{d}_k$. This
15 operation is computationally very complex. A technique
for reducing the computational complexity of implementing
this operation is suggested above with respect to
equations (23) - (33). The present invention improves
this technique.

20 A straightforward approach to this technique
would be to form the FFT of each of the three components
 $\mathbf{S}^T$, $\mathbf{S}$, and $\mathbf{d}_k$, point-wise vector multiply the three
resulting FFTs, and then take the inverse FFT of the

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5 multiplication result. This series of calculations,
however, does not provide an accurate answer because the
matrix S is not a true convolution matrix, and the IFFT
of the triple product, therefore, produces a corrupted
10 result. Figure 3 illustrates the matrix S which is
derived from a vector of symbol decisions of length 2496
(three segments of a VSB field). The first row of the
matrix S contains symbol decision 512 in the first column
down to symbol decision 1 in the last column, the second
15 row of the matrix S contains symbol decision 513 in the
first column down to symbol decision 2 in the last
column, and so on. The portions of the matrix S which
prevent it from being a true convolution matrix are the
triangular areas 40 and 42. The diagonal lines in Figure
3 are parallel.

According to the present invention, and as
shown in Figure 4, matrix S of Figure 3 is decomposed to
form two matrices $\hat{S}$ and F such that the matrix $\hat{S}$ is a
true convolution matrix. Accordingly, the area 44 of the
20 matrix $\hat{S}$ is the same as the area 44 of the matrix S , and
the symbol decisions in the areas 40 and 42 of the matrix
 $\hat{S}$ are set to zero. The symbol decisions in the area 44
of the matrix F are set to zero, and the areas 40 and 42
of the matrix F are the same as the areas 40 and 42 of

SS

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the matrix $\hat{S}$. A new linear system based on $\hat{S}$ and F is then defined which may be solved for the channel estimate h . Advantageously, FFT operations of $\hat{S}$ may be used for implementing the matrix vector multiplication of the

5 conjugate gradient algorithm without corruption because $\hat{S}$ is a true convolution matrix.

More specifically, because the linear system is defined by $y = Sh$, the following equations result:

10
$$y = (\hat{S} + F)h \quad (42)$$

$$y = \hat{S}h + Fh \quad (43)$$

$$y - Fh = \hat{S}h \quad (44)$$

Defining $y - Fh_1 = \hat{y}$, where h_1 is a previous channel

15 estimate derived by any known technique, the new system expression becomes $\hat{y} = \hat{S}h$. This expression can be written as $\hat{S}^T \hat{y} = \hat{S}^T \hat{S}h$. The conjugate gradient algorithm can then be applied to solve for h in the new system as follows:

20

$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S}h_1$$

(2) For $k = 1$ to n , iteratively calculate

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56.

$$(a) \quad d_k = x_k + \beta_k d_{k-1}$$

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad x_{k+1} = x_k - \alpha_k q_{k-1}$$

5 where h_1 may be initialized to any desired value such as 0, and where d_0 , x_0 , β_k , and α_k are as previously defined.

As indicated above following equation (40), step 2(b) includes calculating the expression $q_k = \hat{s}^T \hat{s} d_k$.
10 This multiplication may be conveniently solved by forming the FFT of (i) the matrix $\hat{s}$, (ii) d_k , and (iii) $\hat{s}^T$. The three FFT results are then point-wise vector multiplied, and the inverse FFT (IFFT) is taken of the multiplication result. The first N elements of this IFFT produces q_k .
15 N, for example, can have a value of 512 for a 1024 point FFT/IFFT. However, N can have other values as desired.

Figure 5 shows a system 50 for implementing this technique so as to determine a channel estimate h . The system 50 includes a decoder 52 that decodes the
20 received data y to produce the symbols s . As described above, the decoder 52 may be a trellis decoder. However, the decoder 52 may be other forms of decoders such as slicers. A channel estimator 54 produces a channel estimate h using the techniques described above.

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Modifications of the present invention will occur to those practicing in the art of the present invention. For example, the Fast Fourier Transform (FFT) and Inverse Fast Fourier Transform (IFFT) are used in the present invention as described above. Alternatively, the Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT) can be used instead of the FFT and IFFT.

Accordingly, the description of the present invention is to be construed as illustrative only and is for the purpose of teaching those skilled in the art the best mode of carrying out the invention. The details may be varied substantially without departing from the spirit of the invention, and the exclusive use of all modifications which are within the scope of the appended claims is reserved.

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We claim:

1. A method of processing a received signal y to produce a channel estimate comprising:

5 (a) decoding the received signal y to form data s ;

(b) forming a convolution matrix $\hat{S}$ from the data s ;

(c) forming a matrix F from the data s , wherein
10 the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and,

(d) performing a conjugate gradient algorithm to determine the channel estimate, wherein the conjugate gradient algorithm is based on the received signal y , the
15 matrix $\hat{S}$, and the matrix F .

2. The method of claim 1 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following
20 equation:

$$q_k = \hat{S}^T \hat{S} d_k,$$

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59.

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

3. The method of claim 1 wherein the forming
10 of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,
forming the matrix $\hat{S}$ from the matrix S by
setting certain values of the matrix S to zero;

and wherein the forming of a matrix F from the
15 data s comprises:

forming the matrix F from the matrix S by
setting to zero the values of the matrix S not set to
zero during the forming of the matrix $\hat{S}$.

20 4. The method of claim 3 wherein the
performing of a conjugate gradient algorithm comprises
determining a quantity q_k according to the following
equation:

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$$q_k = \hat{S}^T \hat{S} d_k,$$

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined
5 by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

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5. The equalizer of claim 1 wherein the performing of a conjugate gradient algorithm to determine the channel estimate h comprises performing the following algorithm:

15

$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S} h_1$$

(2) For $k = 1$ to n , iteratively calculate

$$(a) \quad d_k = r_k + \beta_k d_{k-1}$$

20

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad r_{k+1} = r_k - \alpha_k q_{k-1}$$

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where h_1 is an initial value of the channel estimate,

where $\beta_1 = 0$, $\beta_{k \geq 2} = \frac{r_k^T \cdot r_k}{r_{k-1}^T \cdot r_{k-1}}$, where $\alpha_k = \frac{r_k^T \cdot r_k}{d_k \cdot q_k}$, where

$q_k = S^T S d_k$.

5 6. The method of claim 5 wherein the
performing of a conjugate gradient algorithm comprises
determining the quantity q_k by forming a first FFT of the
matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by
forming a third FFT of d_k , by multiplying the first,
10 second, and third FFTs to produce a multiplication
result, and by forming an inverse FFT of the
multiplication result.

7. The method of claim 5 wherein the forming
15 of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,

forming the matrix $\hat{S}$ from the matrix S by
setting certain values of the matrix S to zero;

and wherein the forming of a matrix F from the
20 data s comprises:

forming the matrix F from the matrix S by
setting to zero the values of the matrix S not set to
zero during forming of the matrix $\hat{S}$.

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8. The method of claim 7 wherein the performing of a conjugate gradient algorithm comprises determining the quantity q_k by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

10

9. A method of processing a received signal y comprising:

(a) decoding the received signal y to form data s ;

15 (b) forming a convolution matrix $\hat{S}$ from the data s ;

(c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and,

20 (d) performing a conjugate gradient algorithm based on the received signal y , the matrix $\hat{S}$, and the matrix F .

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10. The method of claim 9 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following equation:

5

$$q_k = \hat{S}^T \hat{S} d_k,$$

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined
10 by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

15

11. The method of claim 9 wherein the forming of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,
forming the matrix $\hat{S}$ from the matrix S by
20 setting certain values of the matrix S to zero;
and wherein the forming of a matrix F from the data s comprises:

forming the matrix F from the matrix S by setting to zero the values of the matrix S not set to zero during forming of the matrix $\hat{S}$.

5 12. The method of claim 11 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following equation:

$$\mathbf{10} \quad \mathbf{g}_k = \hat{\mathbf{S}}^T \hat{\mathbf{S}} \mathbf{d}_k,$$

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined by forming a first FFT of the matrix $\hat{S}$, by forming a

15 second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

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13. A method of processing a received signal
y comprising:

(a) decoding the received signal y to form data
s;

5 (b) forming a convolution matrix $\hat{S}$ from the
data s;

(c) forming a matrix F from the data s, wherein
the matrix F results from forming the matrix $\hat{S}$ as a
convolution matrix; and,

10 (d) performing a conjugate gradient algorithm
based on the received signal y, the matrix $\hat{S}$, and the
matrix F, wherein the conjugate gradient algorithm
includes forming FFTs based on the received signal y, the
matrix $\hat{S}$, and the matrix F, multiplying the FFTs to form
15 a multiplication product, and forming an inverse FFT of
the multiplication product.

14. The method of claim 13 wherein the
performing of a conjugate gradient algorithm comprises
20 determining a quantity q_k according to the following
equation:

$$q_k = \hat{S}^T \hat{S} d_k,$$

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wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

15. The method of claim 13 wherein the forming of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,

forming the matrix $\hat{S}$ from the matrix S by setting certain values of the matrix S to zero;

and wherein the forming of a matrix F from the data s comprises:

forming the matrix F from the matrix S by setting to zero the values of the matrix S not set to zero during forming of the matrix $\hat{S}$.

16. The method of claim 15 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following equation:

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67.

$$q_k = \hat{S}^T \hat{S} d_k,$$

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined
5 by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

10

17. A method of processing a received signal y to produce a channel estimate comprising:

(a) decoding the received signal y to form data s ;

15 (b) forming a convolution matrix $\hat{S}$ from the data s ;

(c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and,

20 (d) performing a conjugate gradient algorithm to determine the channel estimate, wherein the conjugate gradient algorithm is based on the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein the conjugate gradient algorithm includes forming FFTs based on the

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received signal y , the matrix $\hat{S}$, and the matrix F ,
multiplying the FFTs to form a multiplication product,
and forming an inverse FFT of the multiplication product.

5 18. The method of claim 17 wherein the
performing of a conjugate gradient algorithm comprises
determining a quantity q_k according to the following
equation:

10
$$q_k = \hat{S}^T \hat{S} d_k,$$

wherein d_k is dependent upon the received signal y , the
matrix $\hat{S}$, and the matrix F , and wherein q_k is determined
by forming a first FFT of the matrix $\hat{S}$, by forming a
15 second FFT of the matrix $\hat{S}^T$, by forming a third FFT of
 d_k , by multiplying the first, second, and third FFTs to
produce a multiplication result, and by forming an
inverse FFT of the multiplication result.

20 19. The method of claim 17 wherein the forming
of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,
forming the matrix $\hat{S}$ from the matrix S by
setting certain values of the matrix S to zero;

and wherein the forming of a matrix F from the data s comprises:

forming the matrix F from the matrix S by setting to zero the values of the matrix S not set to
5 zero during forming of the matrix $\hat{S}$.

20. The method of claim 19 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following
10 equation:

$$q_k = \hat{S}^T \hat{S} d_k,$$

wherein d_k is dependent upon the received signal y, the
15 matrix $\hat{S}$, and the matrix F, and wherein q_k is determined by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an
20 inverse FFT of the multiplication result.

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21. The equalizer of claim 17 wherein the performing of a conjugate gradient algorithm to determine the channel estimate h comprises performing the following algorithm:

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$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S} h_1$$

(2) For $k = 1$ to n , iteratively calculate

$$(a) \quad d_k = r_k + \beta_k d_{k-1}$$

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$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad r_{k+1} = r_k - \alpha_k q_{k-1}$$

where h_1 is an initial value of the channel estimate,

where $\beta_1 = 0$, $\beta_{k \geq 2} = \frac{r_k^T \cdot r_k}{r_{k-1}^T \cdot r_{k-1}}$, where $\alpha_k = \frac{r_k^T \cdot r_k}{d_k \cdot q_k}$, where

15 $q_k = S^T S d_k.$

22. The method of claim 21 wherein the performing of a conjugate gradient algorithm comprises determining the quantity q_k by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of d_k , by multiplying the first, second, and third FFTs to produce a multiplication

20

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result, and by forming an inverse FFT of the multiplication result.

23. The method of claim 21 wherein the
5 forming of a matrix $\hat{S}$ from the data s comprises:
forming a matrix S from the data s ; and,
forming the matrix $\hat{S}$ from the matrix S by
setting certain values of the matrix S to zero;
and wherein the forming of a matrix F from the
10 data s comprises:
forming the matrix F from the matrix S by
setting to zero the values of the matrix S not set to
zero during forming of the matrix $\hat{S}$.

15 24. The method of claim 23 wherein the
performing of a conjugate gradient algorithm comprises
determining the quantity q_k by forming a first FFT of the
matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by
forming a third FFT of the d_k , by multiplying the first,
20 second, and third FFTs to produce a multiplication
result, and by forming an inverse FFT of the
multiplication result.

Abstract

In forming a channel estimate, a received signal y is decoded to form data s , a convolution matrix $\hat{S}$ is formed from the data s , a matrix F is formed from the data s such that the matrix F results from the forming of the matrix $\hat{S}$ as a convolution matrix, and a conjugate gradient algorithm is performed to determine the channel estimate. The conjugate gradient algorithm is based on the received signal y , the matrix $\hat{S}$, and the matrix F .

Title: Efficient Conjugate Gradient Based Channel Estimator

Inventor: Jin H. Kim and Serdar Ozen

Docket No.: 7240

Serial No.: 10/729,722

Page 1 of 2

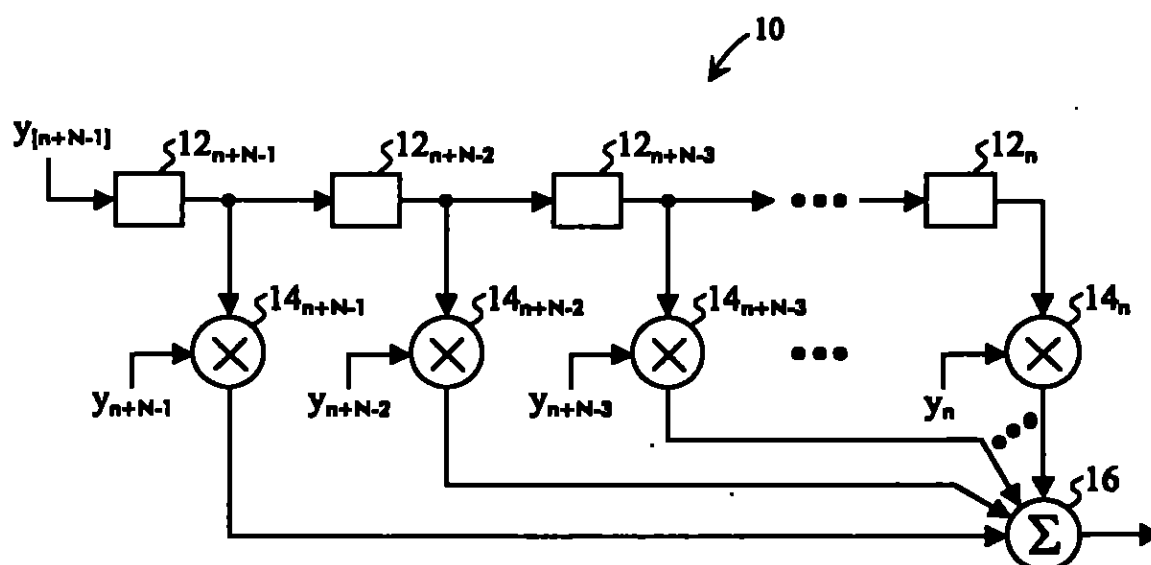


Figure 1

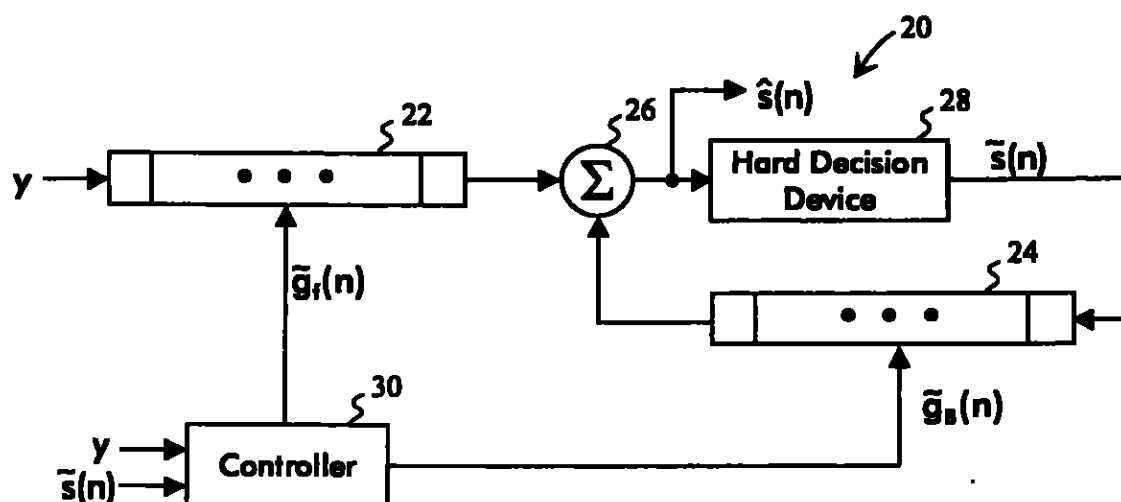


Figure 2

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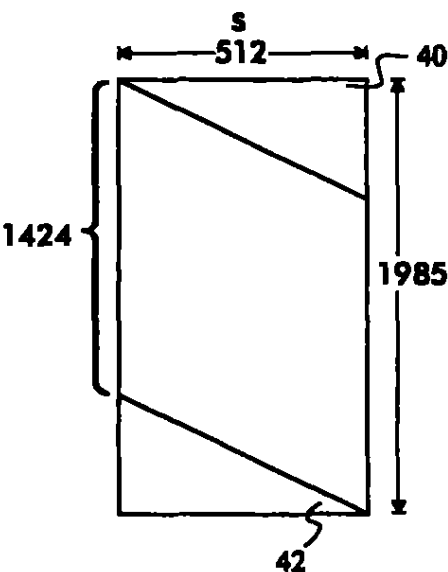


Figure 3

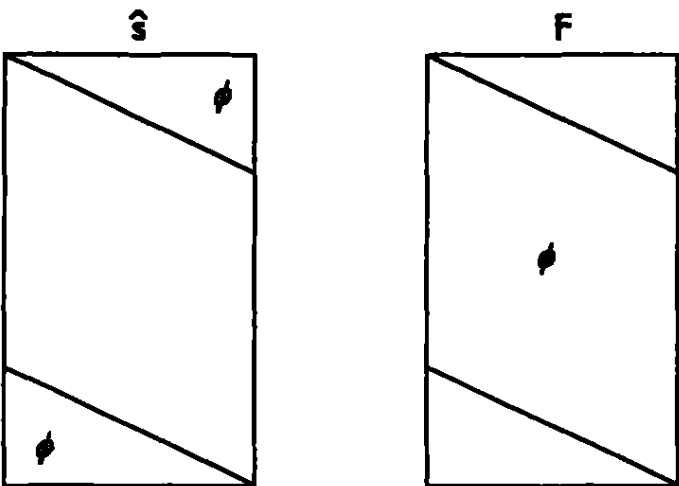


Figure 4

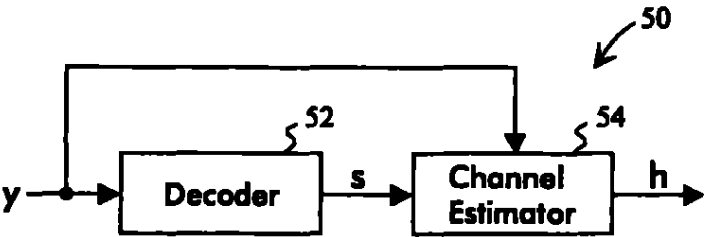


Figure 5

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CONJUGATE GRADIENT BASED CHANNEL ESTIMATOR

Technical Field of the Invention

5 The present invention relates to a conjugate gradient based channel estimator.

Background of the Invention

 Since the adoption of the ATSC digital
10 television (DTV) standard in 1996 in the United States,
there has been an ongoing effort to improve the design of
receivers built for the ATSC DTV signal. The primary
obstacle that faces designers in designing receivers so
that they achieve good reception is the presence of
15 multipath interference in the channel.

 The broadcast television channel is a
relatively severe multipath environment due to a variety
of conditions that are encountered in the channel and at
the receiver. Channels are characterized by impulse
20 responses which may be several hundreds of symbols long,
so that strong interfering signals may arrive at the
receiver both before and after the largest amplitude
signal. In addition, the signal transmitted through the
channel is subject to time varying channel conditions due
25 to the movement of the transmitter and signal reflectors,
airplane flutter, and, for indoor reception, people

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equalizer to channel conditions, the field sync segment of the frame as defined in the ATSC standard may be used as a training sequence for the equalizer. But when equalization is done in the time domain, long equalizers
5 (those having many taps) are required due to the long channel impulse responses that characterize the channel.

The original Grand Alliance receiver used an adaptive decision feedback equalizer (DFE) with 256 taps. This adaptive decision feedback equalizer was adapted to
10 the channel using a standard least mean square (LMS) algorithm, and was trained with the field sync segment of the transmitted frame. The LMS algorithm converges quite slowly and, even with only 256 taps, does not always converge during a single training sequence. Because the
15 field sync segment is transmitted relatively infrequently (about every 260,000 symbols), the total convergence time of this equalizer is quite long if the equalizer only adapts on training symbols prior to convergence.

Therefore, in order to adapt equalizers to
20 follow channel variations that occur between training sequences, the addition of blind and decision directed methods to equalizers has been suggested. However, when implemented in a realistic system, these methods may require several data fields to achieve convergence, and

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As mentioned previously, adaptive equalizers utilizing the least mean square (LMS) algorithm may converge slowly or not at all depending on the channel conditions. Convergence may be adversely affected if the input data auto-correlation matrix has a large eigenvalue spread. Also, if the decision feedback equalizer has not converged before the end of the training sequence, the shape of the objective function may change so that it includes local minima. These local minima may be caused by closed eye channel conditions and decision feedback equalizer error propagation.

The recursive least square (RLS) algorithm is well known to avoid these convergence problems. However, the recursive least square algorithm, in its basic form, requires the computationally intensive inversion of the input data auto-correlation matrix. Lattice based forms of the recursive least square algorithm avoid the need for this matrix inversion. However, the lattice based forms of the recursive least square algorithm are not easily amenable to the advantage of initialization from an initial channel impulse response (CIR) estimate, even though such an initialization may be desirable.

Recent work in reduced rank filtering has connected the multi-stage nested Wiener filter (MSNWF) of

IEEE Transactions on Signal Processing, February 2000.

The mathematical richness of the algorithm relationships leads to a variety of options in the implementation of the basic conjugate gradient algorithm for minimizing a
5 quadratic objective function. These options can be carried over to the adaptive situation to provide a number of options for implementation of the algorithm. One characteristic of the algorithm which may also be exploited is that it works directly with the correlation
10 matrices and vectors. This characteristic makes initialization based on an initial channel impulse response straightforward, assuming information is available for this purpose.

Like lattice based forms of the recursive least
15 square algorithm, the conjugate gradient algorithm does not require the inversion of the input data auto-correlation matrix. Another characteristic of the conjugate gradient algorithm may be exploited by recognizing that the input data auto-correlation matrix
20 is the product of two Toeplitz matrices.

The invention described in U.S. Patent Application Serial No. (7227) uses these characteristics to substantially reduce the computation load in the conjugate gradient algorithm and to even eliminate the

signal y comprises the following: (a) decoding the received signal y to form data s ; (b) forming a convolution matrix $\hat{S}$ from the data s ; (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and, (d) performing a conjugate gradient algorithm based on the received signal y , the matrix $\hat{S}$, and the matrix F .

In accordance with still another aspect of the present invention, a method of processing a received signal y comprises the following: (a) decoding the received signal y to form data s ; (b) forming a convolution matrix $\hat{S}$ from the data s ; (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and, (d) performing a conjugate gradient algorithm based on the received signal y , the matrix $\hat{S}$, and the matrix F , wherein the conjugate gradient algorithm includes forming FFTs based on the received signal y , the matrix $\hat{S}$, and the matrix F , multiplying the FFTs to form a multiplication product, and forming an inverse FFT of the multiplication product.

In accordance with yet another aspect of the present invention, a method of processing a received signal y to produce a channel estimate comprises the

Figure 2 illustrates the decision feedback equalizer according to an embodiment of the present invention;

Figure 3 illustrates a matrix S which is
5 derived from a vector of symbol decisions of length 2496 and is useful in describing the present invention;

Figure 4 illustrates the decomposition of the matrix S to form two matrices $\hat{S}$ and F useful in implementing the present invention; and,

10 Figure 5 shows a system for determining a channel estimate h based on the matrices $\hat{S}$ and F .

Detailed Description

As shown in Figure 1, a filter 10 can be used
15 as an equalizer and includes storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$. For example, N may be 512. The storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$ store corresponding received data $y[n]$, ..., $y[n+N-3]$, $y[n+N-2]$, $y[n+N-1]$. The output of
20 each of the storage registers $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$ is coupled to a corresponding multiplier $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$. Accordingly, the multipliers $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$ receive the stored data from the

The hard decision device 28 (such as a slicer) determines the closest value $\tilde{s}[n]$ for each of the symbols provided as the output $\hat{s}[n]$ of the summer 26. These values $\tilde{s}[n]$ are fed back as inputs to the feedback filter 24. The

5 feedback filter 24 applies tap weights $\hat{g}_b[n]$ to the values $\tilde{s}[n]$ and provides its output to the summer 26. Likewise, the feed forward filter 22 applies tap weights $\hat{g}_f[n]$ to the values $y[n]$ of the received signal and provides its output to the summer 26. The summer 26 sums the output

10 of the feedback filter 24 with the output of the feed forward filter 22 in order to produce the output $\hat{s}[n]$.

The output $\hat{s}[n]$ of the decision feedback equalizer 20 is given by the following equation:

$$\begin{aligned}
 \hat{s}[n] &= \text{Re}\{g_f^H y[n]\} + g_b^T \tilde{s}[n] \\
 &= g_m^T y_n[n] + g_{ni}^T y_i[n] + g_b^T \tilde{s}[n]
 \end{aligned} \tag{1}$$

where $\tilde{s}[n]$ as indicated above is defined to be the symbol estimates provided by the hard decision device 28, where

20 $g_m = \text{Re}\{g_f\}$ is the real part of the tap weights g_f of the feed forward filter 22, where $g_{ni} = \text{Im}\{g_f\}$ is the imaginary part of the tap weights g_f of the feed forward filter 22,

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The equalizer input $\mathbf{y}[n]$ and the feed forward tap weights $\mathbf{g}_f$ are complex values, the feedback tap weights $\mathbf{g}_b$ are real, and the output $\hat{s}[n]$ of the decision feedback equalizer 20 is real. The imaginary part of the output of the feed forward filter 22, $\mathbf{g}_f$, may be ignored and, therefore, not computed.

It is desired to determine the tap weight vector $\tilde{\mathbf{g}}[n]$ so that the error at the output of the decision feedback equalizer 20 is minimized according to the following equations:

$$\tilde{\mathbf{g}}[n] = \arg \min_{\mathbf{g}} \frac{1}{2} E\{s[n] - \hat{s}[n]\}^2 \quad (7)$$

$$\begin{aligned} F(\mathbf{g}) &= E\{s[n] - \hat{s}[n]\}^2 \\ &= \sigma_s^2 - \tilde{\mathbf{g}}^H[n] \mathbf{r}_{sy} - \mathbf{r}_{sy}^H \tilde{\mathbf{g}}[n] + \tilde{\mathbf{g}}^H[n] \mathbf{R}_{yy} \mathbf{g}[n] \end{aligned} \quad (8)$$

15

where σ_s^2 is the symbol variance and is equal to the average symbol energy, $\mathbf{R}_{yy}$ is an autocorrelation matrix, and $\mathbf{r}_{sy}$ is a cross correlation vector. The autocorrelation matrix $\mathbf{R}_{yy}$ and the cross correlation vector $\mathbf{r}_{sy}$ are defined in detail hereinafter. The error function $F(\mathbf{g})$ is minimized by setting its gradient to zero according to the following equation:

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the auto-correlation matrix $R_{yy}[n]$ is given by the following equation:

$$R_{yy}[n] = E[\tilde{y}[n]\tilde{y}^H[n]] \quad (12)$$

5

and the cross correlation vector $r_{sy}[n]$ is given by the following equation:

$$r_{sy}[n] = E[s[n]\tilde{y}^H[n]] \quad (13)$$

10

where $s[n]$ represents the transmitted symbols, E is the expectation operator, and H denotes the Hermitian transpose.

The auto-correlation matrix $R_{yy}[n]$ and the cross correlation vector $r_{sy}[n]$ can be estimated using the forgetting factor method according to the following equations:

$$\begin{aligned} \hat{R}_{yy}[n] &= \sum_{m=0}^{n-1} \gamma^m \tilde{y}[n-m] \tilde{y}^H[n-m] \\ &= \gamma \hat{R}_{yy}[n-1] + \tilde{y}[n] \tilde{y}^H[n] \end{aligned} \quad (14)$$

20

$$\hat{r}_{sy}[n] = \sum_{m=0}^{n-1} \gamma^m s[n-m] \tilde{y}^H[n-m]$$

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$y_R[N_{ts} - N_{fb}], \dots, y_R[N_{ff} - 1]$ are the real part of the
 received data after the training sequence. This setting
 is repeated for $\tilde{y}_r[0]$. Then, $\tilde{y}_r[0] = [y_R^T[0] y_I^T[0]]^T$ is
 constructed as given in equation (2) and $\tilde{y}[0] = [\tilde{y}_r^T[0] \tilde{y}_i^T[0]]^T$
 5 is constructed as given in equation (3) based on the
 initialized vectors $\tilde{y}_r[0]$, $\tilde{y}_i[0]$, and $\tilde{y}[0]$ as given
 above. In an ATSC digital television environment, for
 example, the number of training symbols N_{ts} may be 728,
 the number of feed forward taps N_{ff} may be 512, and the
 10 number of feedback taps N_{fb} may also be 512.

Further, the controller 30 initializes the
 output $\hat{s}[n]$ of the decision feedback equalizer 20 as
 $\hat{s}[0] = \tilde{g}_0^T[0] \tilde{y}[0]$ in accordance with equation (6), the
 controller 30 initializes a gradient $t_a[n]$ as
 15 $t_0[0] = \hat{R}_{yy}[0] \tilde{g}_0[0] - \hat{z}_{ay}[0]$, and the controller 30 sets the
 variable n to 0.

The controller 30 then executes a conjugate
 gradient algorithm once for each received data element,
 where each iteration of the algorithm is initiated upon
 20 the shifting of a new received data element into the feed
 forward filter 22. The parameters of the conjugate
 gradient algorithm as described above are defined in the
 table.

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$$\tilde{g}_1[n] = \tilde{g}_0[n] + \alpha_0[n]b_0[n]$$

$$4. \quad n = n + 1$$

$$\tilde{g}_0[n] = \tilde{g}_1[n - 1]$$

$$t_{-1}[n] = t_0[n - 1]$$

$$5 \quad b_{-1}[n] = b_0[n - 1]$$

5. update input vector $\tilde{y}[n]$ with the next received data element and update $\hat{R}_{yy}[n]$ per equation (14)

6. Calculate output of the decision feedback equalizer 20

$$\hat{s}[n] = \tilde{g}_0^T[n]\tilde{y}[n]$$

7. calculate gradient $t_k[n]$

$$t_0[n] = t_{-1}[n] + \alpha_0[n]|\hat{R}_{yy}[n]b_0[n]|$$

8. Return to step 1

15

The largest of the part of the computations required by this conjugate gradient algorithm is contributed by the matrix-vector multiplications $\hat{R}_{yy}[n]b_0[n]$ of steps 2 and 7. These multiplications

20 require the update of matrix $\hat{R}_{yy}[n]$ from the vector $\tilde{y}[n]$ as in equation (14). This update requires one multiply for each of the N_{taps}^2 matrix elements, a total of N_{taps}^2 multiplies. Then, the matrix-vector multiplication

where dc means don't care. Equation (17) is equivalent to $a[n] = y[n] \otimes b[n]$ where $\otimes$ denotes circular convolution. Circular convolution is equivalent to the following operation:

5

$$\mathcal{Y} = \text{FFT}(\mathbf{y}) \quad (18)$$

$$\mathcal{B} = \text{FFT}(\mathbf{b}) \quad (19)$$

$$\mathcal{A} = \mathcal{Y} \times \mathcal{B} \quad (20)$$

$$\mathbf{a} = \text{IFFT}(\mathcal{A}) \quad (21)$$

10

where FFT in equations (18) and (19) indicates the Fast Fourier Transform operation, IFFT in equation (21) indicates the Inverse Fast Fourier Transform operation, and $\times$ in equation (20) indicates point-wise vector

15 multiplication such that the elements of the resultant vector are given by $Y_0 \times B_0$, $Y_1 \times B_1$, $Y_2 \times B_2$, etc.

An embellishment on the above process involves 0 padding the columns to lengths of 2^N . This 0 padding, as is well known, permits a computationally efficient

20 FFT. Accordingly, the matrix-vector multiplication $\mathbf{a} = \mathbf{Yb}$ can be written with 0 padding in matrix form according to the following equation:

$$\mathbf{v} = \hat{\mathbf{R}}_{\mathbf{y}\mathbf{y}}[n]\mathbf{b}_0[n] = \mathbf{Y}^H \mathbf{Y} \mathbf{b}_0[n] \quad (23)$$

In order to calculate $\mathbf{v}$, the quantity $\mathbf{Y}^H \mathbf{b}_0[n]$ should first
5 be calculated according to the following equations:

$$\mathbf{Y} = \text{FFT}(\mathbf{y}) \quad (24)$$

$$\mathbf{b} = \text{FFT}(\mathbf{b}_0) \quad (25)$$

$$\mathbf{X} = \mathbf{Y} \times \mathbf{b} \quad (26)$$

$$10 \quad \mathbf{x} = \text{IDFT}(\mathbf{X}) \quad (27)$$

$$\mathbf{x}_N = \text{first } N \text{ elements of } \mathbf{x}, \text{ remaining elements} = 0 \quad (28)$$

$$\mathbf{X}_N = \text{FFT}(\mathbf{x}_N) \quad (29)$$

where $\mathbf{x}$ in equation (26) denotes point-wise vector
15 multiplication. Then, $\mathbf{Y} \times \mathbf{X}_N$ is calculated according to
the following equations:

$$\mathbf{Y}' = \text{FFT}(\mathbf{Y}) \quad (30)$$

$$\mathbf{\psi} = \mathbf{Y}' \times \mathbf{X}_N \quad (31)$$

$$20 \quad \mathbf{v} = \text{IDFT}(\mathbf{\psi}) \quad (32)$$

$$\mathbf{v}_N = \text{first } N \text{ elements of } \mathbf{v}, \text{ remaining elements} = 0 \quad (33)$$

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decoder decisions on input symbols to form a long sequence of almost perfectly known symbols. This sequence should have relatively few errors.

The channel impulse response to be estimated is of length $L_h = L_{ha} + L_{hc} + 1$ where L_{ha} is the length of the anti-causal part of the channel impulse response and L_{hc} is the length of the causal part of the channel impulse response.

The reliable trellis decoder decisions on the input symbols is a vector designated herein as $\mathbf{s}$ of length L_s . A Toeplitz matrix $\mathbf{S}$ may be defined based on this vector $\mathbf{s}$ according to the following equation:

$$\mathbf{S} = \begin{bmatrix} s_{L_h-1} & s_{L_h-2} & - & - & - & - & - & s_0 \\ - & s_{L_h-1} & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & s_{L_h-1} & - \\ - & - & - & - & - & - & - & - \\ - & - & - & - & - & s_{L_s-L_h} & - & - \\ s_{L_s-1} & s_{L_s-2} & - & - & - & - & - & s_{L_s-L_h} \end{bmatrix} \quad (34)$$

15

where the elements in the matrix of equation (34) are real and consist of the symbol decisions of vector $\mathbf{s}$.

To ensure an over-determined system of equations, the following inequality is necessary:

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p.

symbols. Normally, the trellis decoder would just make output bit pair decisions, but it can also make equally reliable decisions on the input symbols.

The vector $\mathbf{s}$, for example, may be selected as 3
5 segments ($L_s = 2496$ symbols) long. So, three data
segments may be used to produce a single channel impulse
response estimate update. A new channel impulse response
estimate update can be obtained once per segment by
proceeding in a sliding window manner. Optionally,
10 several consecutive channel impulse response estimate
updates can be averaged in order to further improve
channel impulse response estimate accuracy if necessary.
Of course, this additional step of averaging can be a
problem if the channel impulse response is varying
15 rapidly.

A vector $\mathbf{b}$ with fewer than 3 segments of symbol
decisions may be used, but as stated in inequality (35),
the length of vector $\mathbf{b}$ must be at least twice as long as
the channel impulse response to be estimated. Longer $\mathbf{b}$
20 vectors help to diminish the adverse effect of additive
white Gaussian noise on the channel.

The system of equations represented by equation
(36) may be solved using the conjugate gradient algorithm
in a manner similar to that previously described to solve

a/

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The symbol decisions can be made, for example, by a trellis decoder or a slicer.

The system may be written as $\mathbf{S}^T \mathbf{S} \mathbf{h} = \mathbf{S}^T \mathbf{y}$ by multiplying both sides of equation (37) by $\mathbf{S}^T$. The conjugate gradient algorithm can be used to solve this system for the channel estimate $\mathbf{h}$ according to the following steps:

(1) Calculate the residual vector $\mathbf{r}_1$ representing the system $\mathbf{S}^T \mathbf{y} - \mathbf{S}^T \mathbf{S} \mathbf{h}_1$ as follows:

$$\mathbf{r}_1 = \mathbf{S}^T \mathbf{y} - \mathbf{S}^T \mathbf{S} \mathbf{h}_1 \quad (38)$$

where $\mathbf{r}_1$ is a 512 x 1 vector, and where $\mathbf{h}_1$ is the previous channel estimate which may be initialized to any desired value such as $\mathbf{h}_1 = 0$.

(2) For $k = 1$ to n , iteratively calculate the following:

$$(a) \quad \mathbf{d}_k = \mathbf{r}_k + \beta_k \mathbf{d}_{k-1} \quad (39)$$

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where $\beta_1 = 0$, $\beta_{k \geq 2} = \frac{\mathbf{r}_k^T \cdot \mathbf{r}_k}{\mathbf{r}_{k-1}^T \cdot \mathbf{r}_{k-1}}$, and where $\mathbf{d}_k$ is a 512 x 1

vector;

$$(b) \quad \mathbf{h}_{k+1} = \mathbf{h}_k + \alpha_k \mathbf{d}_k \quad (40)$$

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multiplication result. This series of calculations, however, does not provide an accurate answer because the matrix S is not a true convolution matrix, and the IFFT of the triple product, therefore, produces a corrupted
5 result. Figure 3 illustrates the matrix S which is derived from a vector of symbol decisions of length 2496 (three segments of a VSB field). The first row of the matrix S contains symbol decision 512 in the first column down to symbol decision 1 in the last column, the second
10 row of the matrix S contains symbol decision 513 in the first column down to symbol decision 2 in the last column, and so on. The portions of the matrix S which prevent it from being a true convolution matrix are the triangular areas 40 and 42. The diagonal lines in Figure
15 3 are parallel.

According to the present invention, and as shown in Figure 4, matrix S of Figure 3 is decomposed to form two matrices $\hat{S}$ and F such that the matrix $\hat{S}$ is a true convolution matrix. Accordingly, the area 44 of the
20 matrix $\hat{S}$ is the same as the area 44 of the matrix S , and the symbol decisions in the areas 40 and 42 of the matrix $\hat{S}$ are set to zero. The symbol decisions in the area 44 of the matrix F are set to zero, and the areas 40 and 42 of the matrix F are the same as the areas 40 and 42 of

$$(a) \quad d_k = x_k + \beta_k d_{k-1}$$

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad x_{k+1} = x_k - \alpha_k q_{k-1}$$

5 where h_1 may be initialized to any desired value such as 0, and where d_0 , x_0 , β_k , and α_k are as previously defined.

As indicated above following equation (40), step 2(b) includes calculating the expression $q_k = \hat{s}^T \hat{s} d_k$.
 10 This multiplication may be conveniently solved by forming the FFT of (i) the matrix $\hat{s}$, (ii) d_k , and (iii) $\hat{s}^T$. The three FFT results are then point-wise vector multiplied, and the inverse FFT (IFFT) is taken of the multiplication result. The first N elements of this IFFT produces q_k .
 15 N, for example, can have a value of 512 for a 1024 point FFT/IFFT. However, N can have other values as desired.

Figure 5 shows a system 50 for implementing this technique so as to determine a channel estimate h . The system 50 includes a decoder 52 that decodes the
 20 received data y to produce the symbols s . As described above, the decoder 52 may be a trellis decoder. However, the decoder 52 may be other forms of decoders such as slicers. A channel estimator 54 produces a channel estimate h using the techniques described above.

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We claim:

1. A method of processing a received signal y to produce a channel estimate comprising:
 - 5 (a) decoding the received signal y to form data s ;
 - (b) forming a convolution matrix $\hat{S}$ from the data s ;
 - (c) forming a matrix F from the data s , wherein
 - 10 the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and,
 - (d) performing a conjugate gradient algorithm to determine the channel estimate, wherein the conjugate gradient algorithm is based on the received signal y , the
 - 15 matrix $\hat{S}$, and the matrix F .

2. The method of claim 1 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following
- 20 equation:

$$q_k = \hat{S}^T \hat{S} d_k,$$

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$$q_k = \hat{S}^T \hat{S} d_k,$$

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined
 5 by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

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5. The equalizer of claim 1 wherein the performing of a conjugate gradient algorithm to determine the channel estimate h comprises performing the following algorithm:

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$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S} h_1$$

(2) For $k = 1$ to n , iteratively calculate

$$(a) \quad d_k = r_k + \beta_k d_{k-1}$$

20

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad r_{k+1} = r_k - \alpha_k q_{k-1}$$

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8. The method of claim 7 wherein the performing of a conjugate gradient algorithm comprises determining the quantity q_k by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

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9. A method of processing a received signal y comprising:

- (a) decoding the received signal y to form data s ;
- 15 (b) forming a convolution matrix $\hat{S}$ from the data s ;
- (c) forming a matrix F from the data s , wherein the matrix F results from forming the matrix $\hat{S}$ as a convolution matrix; and,
- 20 (d) performing a conjugate gradient algorithm based on the received signal y , the matrix $\hat{S}$, and the matrix F .

forming the matrix F from the matrix S by setting to zero the values of the matrix S not set to zero during forming of the matrix $\hat{S}$.

5 12. The method of claim 11 wherein the performing of a conjugate gradient algorithm comprises determining a quantity q_k according to the following equation:

$$10 \quad \mathbf{q}_k = \hat{\mathbf{S}}^T \hat{\mathbf{S}} \mathbf{d}_k,$$

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^x$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

wherein d_k is dependent upon the received signal y , the matrix $\hat{S}$, and the matrix F , and wherein q_k is determined by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of the d_k , by multiplying the first, second, and third FFTs to produce a multiplication result, and by forming an inverse FFT of the multiplication result.

15. The method of claim 13 wherein the forming
10 of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,

forming the matrix $\hat{S}$ from the matrix S by
setting certain values of the matrix S to zero;

and wherein the forming of a matrix F from the
15 data s comprises:

forming the matrix F from the matrix S by
setting to zero the values of the matrix S not set to
zero during forming of the matrix $\hat{S}$.

20 16. The method of claim 15 wherein the
performing of a conjugate gradient algorithm comprises
determining a quantity q_k according to the following
equation:

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received signal y , the matrix $\hat{S}$, and the matrix F ,
multiplying the FFTs to form a multiplication product,
and forming an inverse FFT of the multiplication product.

- 5 18. The method of claim 17 wherein the
performing of a conjugate gradient algorithm comprises
determining a quantity q_k according to the following
equation:

10 $q_k = \hat{S}^T \hat{S} d_k,$

wherein d_k is dependent upon the received signal y , the
matrix $\hat{S}$, and the matrix F , and wherein q_k is determined
by forming a first FFT of the matrix $\hat{S}$, by forming a
15 second FFT of the matrix $\hat{S}^T$, by forming a third FFT of
 d_k , by multiplying the first, second, and third FFTs to
produce a multiplication result, and by forming an
inverse FFT of the multiplication result.

- 20 19. The method of claim 17 wherein the forming
of a matrix $\hat{S}$ from the data s comprises:

forming a matrix S from the data s ; and,
forming the matrix $\hat{S}$ from the matrix S by
setting certain values of the matrix S to zero;

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21. The equalizer of claim 17 wherein the performing of a conjugate gradient algorithm to determine the channel estimate h comprises performing the following algorithm:

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$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S} h_1$$

(2) For $k = 1$ to n , iteratively calculate

10

$$(a) \quad d_k = r_k + \beta_k d_{k-1}$$

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad r_{k+1} = r_k - \alpha_k q_k$$

where h_1 is an initial value of the channel estimate,

where $\beta_1 = 0$, $\beta_{k \geq 2} = \frac{r_k^T \cdot r_k}{r_{k-1}^T \cdot r_{k-1}}$, where $\alpha_k = \frac{r_k^T \cdot r_k}{d_k \cdot q_k}$, where

$$15 \quad q_k = S^T S d_k.$$

22. The method of claim 21 wherein the performing of a conjugate gradient algorithm comprises determining the quantity q_k by forming a first FFT of the matrix $\hat{S}$, by forming a second FFT of the matrix $\hat{S}^T$, by forming a third FFT of d_k , by multiplying the first, second, and third FFTs to produce a multiplication

20

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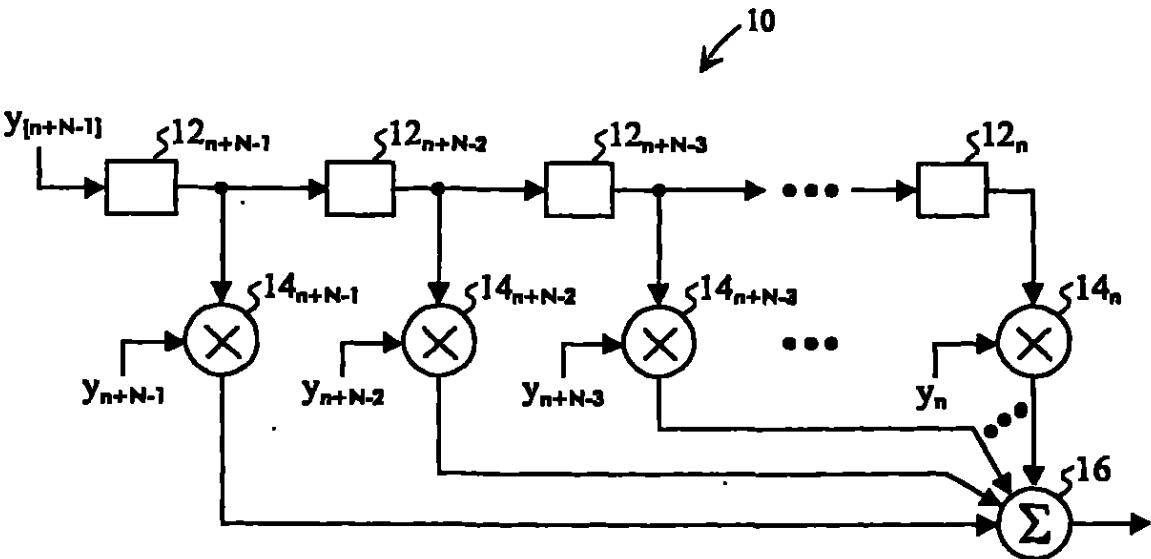


Figure 1

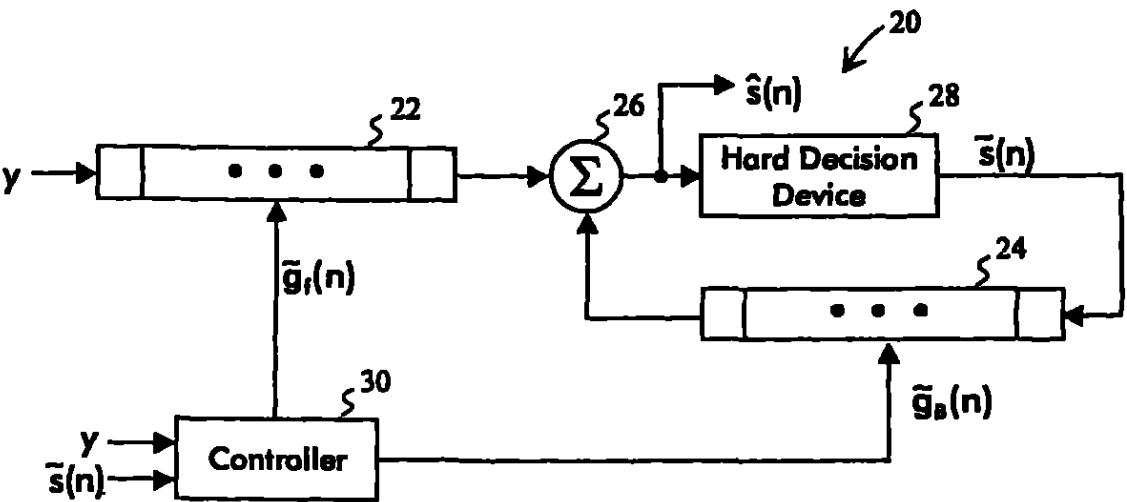


Figure 2

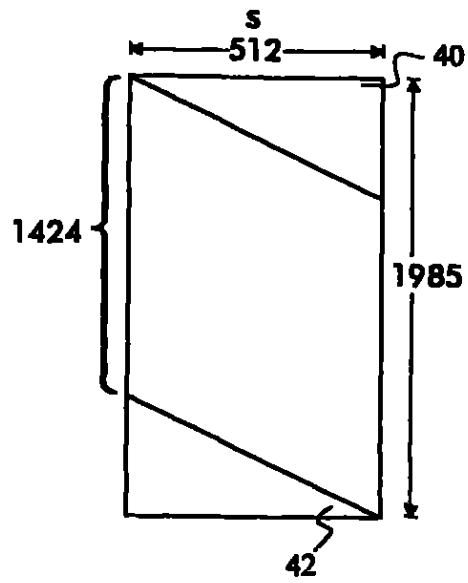


Figure 3

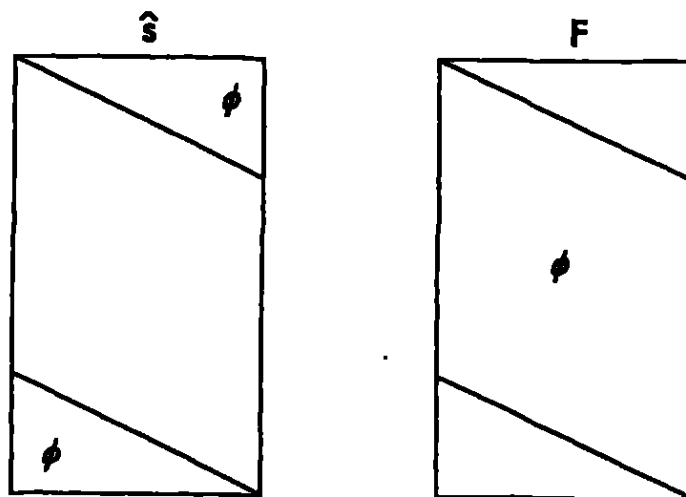


Figure 4

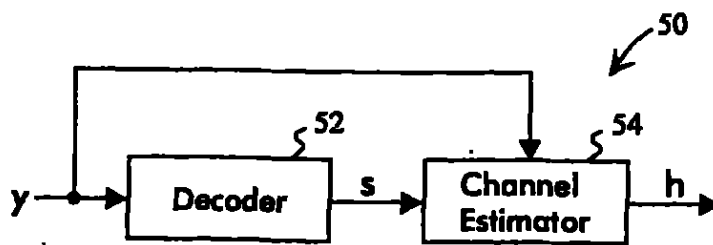


Figure 5

INTERNATIONAL SEARCH REPORT

Application No

/US2004/034642

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 H04L25/02

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7 H04L G06F

Documents searched other than minimum documentation to the extent that such documents are included in the fields searched

Literature data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, INSPEC, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 6 647 078 B1 (THOMAS TIMOTHY A ET AL) 11 November 2003 (2003-11-11) abstract column 9, paragraph 2	1-24
A	WO 03/036891 A (ZENITH ELECTRONICS CORPORATION) 1 May 2003 (2003-05-01) abstract claims 1,10	1-24
	----- -/--	

☒ Further documents are listed in the continuation of box C.

☒ Patent family members are listed in annex.

* Special categories of cited documents:

- *A* document defining the general state of the art which is not considered to be of particular relevance
- *E* earlier document but published on or after the international filing date
- *L* document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
- *O* document referring to an oral disclosure, use, exhibition or other means
- *P* document published prior to the international filing date but later than the priority date claimed

- *T* later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
- *X* document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
- *Y* document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
- *A* document member of the same patent family

Date of the actual completion of the international search

9 February 2005

Date of mailing of the international search report

16/02/2005

Name and mailing address of the ISA

European Patent Office, P.B. 5818 Patentplan 2
NL - 2200 HV Rijswijk
Tel. (+31-70) 340-2040, Tx. 31 651 epo nl,
Fax (+31-70) 340-3018

Authorized officer

Koukourlis, S

INTERNATIONAL SEARCH REPORT

Application No
T/US2004/034642

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT		
Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	SARKAR T K ET AL: "ACCURATE AND EFFICIENT SOLUTION OF HANKEL MATRIX SYSTEMS BY FFT AND THE CONJUGATE GRADIENT METHODS" INTERNATIONAL CONFERENCE ON ACOUSTICS, SPEECH & SIGNAL PROCESSING. ICASSP. DALLAS, APRIL 6 - 9, 1987, NEW YORK, IEEE, US, vol. 3 CONF. 12, April 1987 (1987-04), pages 1835-1838, XP000758077 the whole document	1-24
A	SONI R A ET AL: "Channel equalization with adaptive filtering and the preconditioned conjugate gradient algorithm" CIRCUITS AND SYSTEMS, 1997. ISCAS '97., PROCEEDINGS OF 1997 IEEE INTERNATIONAL SYMPOSIUM ON HONG KONG 9-12 JUNE 1997, NEW YORK, NY, USA, IEEE, US, vol. 4, 9 June 1997 (1997-06-09), pages 2284-2287, XP010236182 ISBN: 0-7803-3583-X abstract	1-24
A	ZOLTOWSKI M D ET AL: "Conjugate gradient based multichannel decision feedback equalization for digital television" 2002 IEEE SENSOR ARRAY AND MULTICHANNEL SIGNAL PROCESSING WORKSHOP PROCEEDINGS (CAT. NO.02EX593) IEEE PISCATAWAY, NJ, USA, 4 August 2002 (2002-08-04), pages 599-603, XP010635821 ISBN: 0-7803-7551-3 abstract	1-24

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INTERNATIONAL SEARCH REPORT

Information on patent family members

Application No

r/US2004/034642

Patent document cited in search report		Publication date	Patent family member(s)	Publication date
US 6647078	B1	11-11-2003	NONE	
WO 03036891	A	01-05-2003	US 2003099309 A1	29-05-2003
			WO 03036891 A2	01-05-2003

Form PCT/ISA/210 (patent family annex) (January 2004)

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From the INTERNATIONAL SEARCHING AUTHORITY

PCT

To:

SCHIFF HARDIN LLP
Attn. Joike, Trevor B.
6600 Sears Tower
Chicago, IL 60606
UNITED STATES OF AMERICA

NOTIFICATION OF TRANSMITTAL OF
THE INTERNATIONAL SEARCH REPORT AND
THE WRITTEN OPINION OF THE INTERNATIONAL
SEARCHING AUTHORITY, OR THE DECLARATION

(PCT Rule 44.1)

Date of mailing (day/month/year) 16/02/2005	
Applicant's or agent's file reference FP03, 0369-WO	FOR FURTHER ACTION See paragraphs 1 and 4 below
International application No. PCT/US2004/034642	International filing date (day/month/year) 18/10/2004
Applicant ZENITH ELECTRONICS CORPORATION	

1. ☒ The applicant is hereby notified that the international search report and the written opinion of the International Searching Authority have been established and are transmitted herewith.

Filing of amendments and statement under Article 19:

The applicant is entitled, if he so wishes, to amend the claims of the International Application (see Rule 46):

When? The time limit for filing such amendments is normally 2 months from the date of transmittal of the International Search Report; however, for more details, see the notes on the accompanying sheet.

Where? Directly to the International Bureau of WIPO, 34 chemin des Colombettes
1211 Geneva 20, Switzerland, Facsimile No.: (41-22) 740.14.35

For more detailed instructions, see the notes on the accompanying sheet.

2. ☐ The applicant is hereby notified that no international search report will be established and that the declaration under Article 17(2)(a) to that effect and the written opinion of the International Searching Authority are transmitted herewith.
3. ☐ With regard to the protest against payment of (an) additional fee(s) under Rule 40.2, the applicant is notified that:

- ☐ the protest together with the decision thereon has been transmitted to the International Bureau together with the applicant's request to forward the texts of both the protest and the decision thereon to the designated Offices.
- ☐ no decision has been made yet on the protest; the applicant will be notified as soon as a decision is made.

4. Reminders

Shortly after the expiration of 18 months from the priority date, the international application will be published by the International Bureau. If the applicant wishes to avoid or postpone publication, a notice of withdrawal of the international application, or of the priority claim, must reach the International Bureau as provided in Rules 90bis.1 and 90bis.3, respectively, before the completion of the technical preparations for international publication.

The applicant may submit comments on an informal basis on the written opinion of the International Searching Authority to the International Bureau. The International Bureau will send a copy of such comments to all designated Offices unless an international preliminary examination report has been or is to be established. These comments would also be made available to the public but not before the expiration of 30 months from the priority date.

Within 18 months from the priority date, but only in respect of some designated Offices, a demand for international preliminary examination must be filed if the applicant wishes to postpone the entry into the national phase until 30 months from the priority date (in some Offices even later); otherwise, the applicant must, within 20 months from the priority date, perform the prescribed acts for entry into the national phase before those designated Offices.

In respect of other designated Offices, the time limit of 30 months (or later) will apply even if no demand is filed within 19 months.

See the Annex to Form PCT/IB/301 and, for details about the applicable time limits, Office by Office, see the *PCT Applicant's Guide*, Volume II, National Chapters and the WIPO Internet site.

Name and mailing address of the International Searching Authority  European Patent Office, P.B. 5818 Patentlaan 2 NL-2280 HV Rijswijk Tel. (+31-70) 340-2040, Tx. 31 651 epo nl, Fax: (+31-70) 340-3016	Authorized officer Carole Emery
--	--

These Notes are intended to give the basic instructions concerning the filing of amendments under article 19. The Notes are based on the requirements of the Patent Cooperation Treaty, the Regulations and the Administrative Instructions under that Treaty. In case of discrepancy between these Notes and those requirements, the latter are applicable. For more detailed information, see also the PCT Applicant's Guide, a publication of WIPO.

In these Notes, "Article", "Rule", and "Section" refer to the provisions of the PCT, the PCT Regulations and the PCT Administrative Instructions respectively.

INSTRUCTIONS CONCERNING AMENDMENTS UNDER ARTICLE 19

The applicant has, after having received the international search report, one opportunity to amend the claims of the international application. It should however be emphasized that, since all parts of the international application (claims, description and drawings) may be amended during the international preliminary examination procedure, there is usually no need to file amendments of the claims under Article 19 except where, e.g. the applicant wants the letter to be published for the purposes of provisional protection or has another reason for amending the claims before international publication. Furthermore, it should be emphasized that provisional protection is available in some States only.

What parts of the international application may be amended?

Under Article 19, only the claims may be amended.

During the international phase, the claims may also be amended (or further amended) under Article 34 before the International Preliminary Examining Authority. The description and drawings may only be amended under Article 34 before the International Examining Authority.

Upon entry into the national phase, all parts of the international application may be amended under Article 28 or, where applicable, Article 41.

When?

Within 2 months from the date of transmittal of the international search report or 16 months from the priority date, whichever time limit expires later. It should be noted, however, that the amendments will be considered as having been received on time if they are received by the International Bureau after the expiration of the applicable time limit but before the completion of the technical preparations for international publication (Rule 46.1).

Where not to file the amendments?

The amendments may only be filed with the International Bureau and not with the receiving Office or the International Searching Authority (Rule 46.2).

Where a demand for international preliminary examination has been/is filed, see below.

How?

Either by cancelling one or more entire claims, by adding one or more new claims or by amending the text of one or more of the claims as filed.

A replacement sheet must be submitted for each sheet of the claims which, on account of an amendment or amendments, differs from the sheet originally filed.

All the claims appearing on a replacement sheet must be numbered in Arabic numerals. Where a claim is cancelled, no renumbering of the other claims is required. In all cases where claims are renumbered, they must be renumbered consecutively (Administrative Instructions, Section 205(b)).

The amendments must be made in the language in which the international application is to be published.

What documents must/may accompany the amendments?

Letter (Section 205(b)):

The amendments must be submitted with a letter.

The letter will not be published with the international application and the amended claims. It should not be confused with the "Statement under Article 19(1)" (see below, under "Statement under Article 19(1)").

The letter must be in English or French, at the choice of the applicant. However, if the language of the international application is English, the letter must be in English; if the language of the international application is French, the letter must be in French.

The letter must indicate the differences between the claims as filed and the claims as amended. It must, in particular, indicate, in connection with each claim appearing in the international application (it being understood that identical indications concerning several claims may be grouped), whether

- (i) the claim is unchanged;
- (ii) the claim is cancelled;
- (iii) the claim is new;
- (iv) the claim replaces one or more claims as filed;
- (v) the claim is the result of the division of a claim as filed.

The following examples illustrate the manner in which amendments must be explained in the accompanying letter:

1. [Where originally there were 48 claims and after amendment of some claims there are 51]:
"Claims 1 to 29, 31, 32, 34, 35, 37 to 48 replaced by amended claims bearing the same numbers; claims 30, 33 and 36 unchanged; new claims 49 to 51 added."
2. [Where originally there were 15 claims and after amendment of all claims there are 11]:
"Claims 1 to 15 replaced by amended claims 1 to 11."
3. [Where originally there were 14 claims and the amendments consist in cancelling some claims and in adding new claims]:
"Claims 1 to 6 and 14 unchanged; claims 7 to 13 cancelled; new claims 15, 16 and 17 added." or
"Claims 7 to 13 cancelled; new claims 15, 16 and 17 added; all other claims unchanged."
4. [Where various kinds of amendments are made]:
"Claims 1-10 unchanged; claims 11 to 13, 18 and 19 cancelled; claims 14, 15 and 16 replaced by amended claim 14; claim 17 subdivided into amended claims 15, 16 and 17; new claims 20 and 21 added."

"Statement under article 19(1)" (Rule 48.4)

The amendments may be accompanied by a statement explaining the amendments and indicating any impact that such amendments might have on the description and the drawings (which cannot be amended under Article 19(1)).

The statement will be published with the international application and the amended claims.

It must be in the language in which the international application is to be published.

It must be brief, not exceeding 500 words if in English or if translated into English.

It should not be confused with and does not replace the letter indicating the differences between the claims as filed and as amended. It must be filed on a separate sheet and must be identified as such by a heading, preferably by using the words "Statement under Article 19(1)."

It may not contain any disparaging comments on the international search report or the relevance of citations contained in that report. Reference to citations, relevant to a given claim, contained in the international search report may be made only in connection with an amendment of that claim.

Consequence if a demand for international preliminary examination has already been filed

If, at the time of filing any amendments under Article 19, a demand for international preliminary examination has already been submitted, the applicant must preferably, at the same time of filing the amendments with the International Bureau, also file a copy of such amendments with the International Preliminary Examining Authority (see Rule 62.2(a), first sentence).

Consequence with regard to translation of the international application for entry into the national phase

The applicant's attention is drawn to the fact that, where upon entry into the national phase, a translation of the claims as amended under Article 19 may have to be furnished to the designated/elected Offices, instead of, or in addition to, the translation of the claims as filed.

For further details on the requirements of each designated/elected Office, see Volume II of the PCT Applicant's Guide.

INTERNATIONAL SEARCH REPORT

(PCT Article 18 and Rules 43 and 44)

Applicant's or agent's file reference FP03, 0369-WO	FOR FURTHER ACTION see Form PCT/ISA/220 as well as, where applicable, item 5 below.	
International application No. PCT/US2004/034642	International filing date (day/month/year) 18/10/2004	(Earliest) Priority Date (day/month/year) 05/12/2003
Applicant ZENITH ELECTRONICS CORPORATION		

This International Search Report has been prepared by this International Searching Authority and is transmitted to the applicant according to Article 18. A copy is being transmitted to the International Bureau.

This International Search Report consists of a total of 4 sheets.

☒ It is also accompanied by a copy of each prior art document cited in this report.

1. Basis of the report

a. With regard to the language, the international search was carried out on the basis of the international application in the language in which it was filed, unless otherwise indicated under this item.

☐ The international search was carried out on the basis of a translation of the international application furnished to this Authority (Rule 23.1(b)).

b. ☐ With regard to any nucleotide and/or amino acid sequence disclosed in the international application, see Box No. I.

2. ☐ Certain claims were found unsearchable (See Box II).

3. ☐ Unity of invention is lacking (see Box III).

4. With regard to the title,

☐ the text is approved as submitted by the applicant.

☒ the text has been established by this Authority to read as follows:

CONJUGATE GRADIENT BASED CHANNEL ESTIMATOR

5. With regard to the abstract,

☒ the text is approved as submitted by the applicant.

☐ the text has been established, according to Rule 38.2(b), by this Authority as it appears in Box No. IV. The applicant may, within one month from the date of mailing of this international search report, submit comments to this Authority.

6. With regard to the drawings,

a. the figure of the drawings to be published with the abstract is Figure No. 4

☐ as suggested by the applicant.

☐ as selected by this Authority, because the applicant failed to suggest a figure.

☒ as selected by this Authority, because this figure better characterizes the invention.

b. ☐ none of the figures is to be published with the abstract.

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7 H04L25/02

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)

IPC 7 H04L 606F

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data bases consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, INSPEC, WPI Data

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 6 647 078 B1 (THOMAS TIMOTHY A ET AL) 11 November 2003 (2003-11-11) abstract column 9, paragraph 2	1-24
A	WO 03/036891 A (ZENITH ELECTRONICS CORPORATION) 1 May 2003 (2003-05-01) abstract claims 1,10	1-24
	----- -/-	



Further documents are listed in the continuation of box C.



Patent family members are listed in annex.

*** Special categories of cited documents:**

- "A" document defining the general state of the art which is not considered to be of particular relevance
- "E" earlier document but published on or after the international filing date
- "L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
- "O" document referring to an oral disclosure, use, exhibition or other means
- "P" document published prior to the international filing date but later than the priority date claimed

- "T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
- "X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
- "Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art.
- "A" document member of the same patent family

Date of the actual completion of the international search

9 February 2005

Date of mailing of the international search report

16/02/2005

Name and mailing address of the ISA

European Patent Office, P.O. 5818 Patentplan 2
NL - 2280 HV Rijswijk
Tel. (+31-70) 340-2040, Tx. 31 651 epo nl,
Fax: (+31-70) 340-3016

Authorized officer

Koukourlis, S

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT		
Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	SARKAR T K ET AL: "ACCURATE AND EFFICIENT SOLUTION OF HANKEL MATRIX SYSTEMS BY FFT AND THE CONJUGATE GRADIENT METHODS" INTERNATIONAL CONFERENCE ON ACOUSTICS, SPEECH & SIGNAL PROCESSING. ICASSP. DALLAS, APRIL 6 - 9, 1987, NEW YORK, IEEE, US, vol. VOL. 3 CONF. 12, April 1987 (1987-04), pages 1835-1838, XP000758077 the whole document	1-24
A	SONI R A ET AL: "Channel equalization with adaptive filtering and the preconditioned conjugate gradient algorithm" CIRCUITS AND SYSTEMS, 1997. ISCAS '97., PROCEEDINGS OF 1997 IEEE INTERNATIONAL SYMPOSIUM ON HONG KONG 9-12 JUNE 1997, NEW YORK, NY, USA, IEEE, US, vol. 4, 9 June 1997 (1997-06-09), pages 2284-2287, XP010236182 ISBN: 0-7803-3583-X abstract	1-24
A	ZOLTOWSKI M D ET AL: "Conjugate gradient based multichannel decision feedback equalization for digital television" 2002 IEEE SENSOR ARRAY AND MULTICHANNEL SIGNAL PROCESSING WORKSHOP PROCEEDINGS (CAT. NO.02EX593) IEEE PISCATAWAY, NJ, USA, 4 August 2002 (2002-08-04), pages 599-603, XP010635821 ISBN: 0-7803-7551-3 abstract	1-24

INTERNATIONAL SEARCH REPORT

Information on patent family members

International Application No

PCT/US2004/034642

111

Patent document cited in search report	Publication date	Patent family member(s)	Publication date
US 6647078	B1	11-11-2003	NONE
WO 03036891	A	01-05-2003	US 2003099309 A1 29-05-2003
		WO 03036891 A2	01-05-2003

From the
INTERNATIONAL SEARCHING AUTHORITY

PCT

To:

see form PCT/ISA/220

WRITTEN OPINION OF THE
INTERNATIONAL SEARCHING AUTHORITY
(PCT Rule 43bis.1)

Date of mailing
(day/month/year) see form PCT/ISA/210 (second sheet)

Applicant's or agent's file reference
see form PCT/ISA/220

FOR FURTHER ACTION
See paragraph 2 below

International application No.
PCT/US2004/034642

International filing date (day/month/year)
18.10.2004

Priority date (day/month/year)
05.12.2003

International Patent Classification (IPC) or both national classification and IPC
H04L25/02

Applicant
ZENITH ELECTRONICS CORPORATION

1. This opinion contains indications relating to the following items:

- ☒ Box No. I Basis of the opinion
- ☐ Box No. II Priority
- ☒ Box No. III Non-establishment of opinion with regard to novelty, inventive step and industrial applicability
- ☐ Box No. IV Lack of unity of invention
- ☐ Box No. V Reasoned statement under Rule 43bis.1(a)(i) with regard to novelty, inventive step or industrial applicability; citations and explanations supporting such statement
- ☐ Box No. VI Certain documents cited
- ☐ Box No. VII Certain defects in the international application
- ☐ Box No. VIII Certain observations on the international application

2. FURTHER ACTION

If a demand for international preliminary examination is made, this opinion will usually be considered to be a written opinion of the International Preliminary Examining Authority ("IPEA"). However, this does not apply where the applicant chooses an Authority other than this one to be the IPEA and the chosen IPEA has notified the International Bureau under Rule 56.1bis(b) that written opinions of this International Searching Authority will not be so considered.

If this opinion is, as provided above, considered to be a written opinion of the IPEA, the applicant is invited to submit to the IPEA a written reply together, where appropriate, with amendments, before the expiration of three months from the date of mailing of Form PCT/ISA/220 or before the expiration of 22 months from the priority date, whichever expires later.

For further options, see Form PCT/ISA/220.

3. For further details, see notes to Form PCT/ISA/220.

Name and mailing address of the ISA:



European Patent Office - P.B. 5818 Patentlaan 2
NL-2280 HV Rijswijk - Pays Bas
Tel. +31 70 340 - 2040 Tx: 31 651 epo nl
Fax: +31 70 340 - 3016

Authorized Officer

Koukourlis, S

Telephone No. +31 70 340-2285



**WRITTEN OPINION OF THE
INTERNATIONAL SEARCHING AUTHORITY**

113
International application No.
PCT/US2004/034642

Box No. 1 Basis of the opinion

1. With regard to the language, this opinion has been established on the basis of the international application in the language in which it was filed, unless otherwise indicated under this item.
☐ This opinion has been established on the basis of a translation from the original language into the following language , which is the language of a translation furnished for the purposes of international search (under Rules 12.3 and 23.1(b)).
2. With regard to any nucleotide and/or amino acid sequence disclosed in the international application and necessary to the claimed invention, this opinion has been established on the basis of:
 - a. type of material:
☐ a sequence listing
☐ table(s) related to the sequence listing
 - b. format of material:
☐ in written format
☐ in computer readable form
 - c. time of filing/furnishing:
☐ contained in the international application as filed.
☐ filed together with the international application in computer readable form.
☐ furnished subsequently to this Authority for the purposes of search.
3. ☐ In addition, in the case that more than one version or copy of a sequence listing and/or table relating thereto has been filed or furnished, the required statements that the information in the subsequent or additional copies is identical to that in the application as filed or does not go beyond the application as filed, as appropriate, were furnished.
4. Additional comments:

Box No. III Non-establishment of opinion with regard to novelty, inventive step and industrial applicability

The questions whether the claimed invention appears to be novel, to involve an inventive step (to be non obvious), or to be industrially applicable have not been examined in respect of:

- ☐ the entire international application,
- ☒ claims Nos. 1-24

because:

- ☐ the said international application, or the said claims Nos. relate to the following subject matter which does not require an international preliminary examination (*specify*):
- ☒ the description, claims or drawings (*indicate particular elements below*) or said claims Nos. 1-24 are so unclear that no meaningful opinion could be formed (*specify*):
see separate sheet
- ☐ the claims, or said claims Nos. are so inadequately supported by the description that no meaningful opinion could be formed.
- ☐ no international search report has been established for the whole application or for said claims Nos.
- ☐ the nucleotide and/or amino acid sequence listing does not comply with the standard provided for in Annex C of the Administrative Instructions in that:
 - the written form ☐ has not been furnished
 - ☐ does not comply with the standard
 - the computer readable form ☐ has not been furnished
 - ☐ does not comply with the standard
- ☐ the tables related to the nucleotide and/or amino acid sequence listing, if in computer readable form only, do not comply with the technical requirements provided for in Annex C-bis of the Administrative Instructions.
- ☐ See separate sheet for further details

Re Item III

Non-establishment of opinion with regard to novelty, inventive step and industrial applicability

Reference is made to the following documents:

- D1: US-B1-6 647 078 (THOMAS TIMOTHY A ET AL) 11 November 2003 (2003-11-11)
- D2: WO 03/036891 A (ZENITH ELECTRONICS CORPORATION) 1 May 2003 (2003-05-01)
- D3: SARKAR T K ET AL: "ACCURATE AND EFFICIENT SOLUTION OF HANKEL MATRIX SYSTEMS BY FFT AND THE CONJUGATE GRADIENT METHODS" INTERNATIONAL CONFERENCE ON ACOUSTICS, SPEECH & SIGNAL PROCESSING. ICASSP. DALLAS, APRIL 6 - 9, 1987, NEW YORK, IEEE, US, vol. VOL. 3 CONF. 12, April 1987 (1987-04), pages 1835-1838, XP000758077
- D4: SONI R A ET AL: "Channel equalization with adaptive filtering and the preconditioned conjugate gradient algorithm" CIRCUITS AND SYSTEMS, 1997. ISCAS '97., PROCEEDINGS OF 1997 IEEE INTERNATIONAL SYMPOSIUM ON HONG KONG 9-12 JUNE 1997, NEW YORK, NY, USA, IEEE, US, vol. 4, 9 June 1997 (1997-06-09), pages 2284-2287, XP010236182 ISBN: 0-7803-3583-X
- D5: ZOLTOWSKI M D ET AL: "Conjugate gradient based multichannel decision feedback equalization for digital television" 2002 IEEE SENSOR ARRAY AND MULTICHANNEL SIGNAL PROCESSING WORKSHOP PROCEEDINGS (CAT. NO.02EX593) IEEE PISCATAWAY, NJ, USA, 4 August 2002 (2002-08-04), pages 599-603, XP010635821 ISBN: 0-7803-7551-3
1. The application does not meet the requirements of Article 6 PCT, because claims 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23 are not clear.
- 1.1 The term *convolution matrix* used in the independent claims is vague and unclear, because the quantities that are convolved are not defined. Thus, it leaves the reader in doubt as to the meaning of the technical features to which it refers, thereby ren-

dering the definition of the subject-matter of said claims unclear.

- 1.2 From the wording of the steps (b) and (c), it is not clear what the difference between the two matrices $\hat{S}$ and F is; they both seem to be convolution matrices *formed from* the data s .
- 1.3 It is clear from the description that, amongst others, the features of claim 3 are essential to the definition of the invention. However, claim 3 only states that *certain values* of the matrix S are set to zero, without defining which those values are.

The problem to which the invention relates, i.e. implementing the matrix-vector multiplication involved in the conjugate gradient algorithm without corruption (see description page 34), cannot be solved without:

- the features relating to forming the FFT of matrix $\hat{S}$ (see step (d) of claim 13);
- the features of claim 3; and
- the features related to decomposing the matrix S to matrices $\hat{S}$ and F in the way described on page 33 and figures 3 and 4 (i.e. setting the upper and lower triangular areas of matrix S to zero, thereby producing matrix $\hat{S}$, and keeping the values in the upper and lower triangular areas of matrix S while setting the remaining values to zero, thereby producing matrix F).

Since the independent claims do not contain these features they do not meet the requirement following from Article 6 PCT taken in combination with Rule 6.3(b) PCT that any independent claim must contain all the technical features essential to the definition of the invention.

- 1.4 Claims 5 and 21 claim an equalizer while they depend on a method. This renders their subject-matter unclear.
- 1.5 The plurality of independent claims makes it difficult to determine the matter for which protection is sought, and places an undue burden on others seeking to establish the extent of the protection. Hence, the requirements of Article 6 PCT (conciseness) are not met.

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- 1.6 The vague and imprecise statement in the description on page 36, last paragraph referring to the *spirit of the invention* implies that the subject-matter for which protection is sought may be different to that defined by the claims, thereby resulting in lack of clarity (Article 6 PCT) when used to interpret them.
 2. In view of the objections mentioned-above, a reasoned statement with regard to novelty, inventive step and industrial applicability will be given with respect to a claim which would include all the technical features which are essential to the definition of the invention.

Document D3 is regarded as being the closest prior art, and shows (see pages 1836-1837) using FFT in order to calculate efficiently matrix-vector products involved in the conjugate gradient algorithm.

The invention as claimed by a claim containing all the essential features, would differ from the disclosure of this document in the way of decomposing (see point 1.3 above) the matrix of which the FFT is to be calculated.

The problem to be solved by the present invention would then be regarded as implementing the matrix-vector multiplication of the conjugate gradient algorithm without corruption.

The solution to this problem would be considered new (Article 33(2) PCT) and involving an inventive step (Article 33(3) PCT) because, although using the gradient algorithm to determine a channel estimate (see D1, D2) or for processing a received signal for other purposes (e.g. channel equalisation, see D4, D5) is known from the prior art, none of the prior art documents discloses or renders obvious the use of the specific matrix decomposition.

As the method of the invention would be used in processing a received signal, it would be industrially applicable (Article 33(4) PCT).



(12) **United States Patent**
Thomas et al:

(10) Patent No.: **US 6,647,078 B1**
(45) Date of Patent: **Nov. 11, 2003**

- (54) **METHOD AND DEVICE FOR MULTI-USER FREQUENCY-DOMAIN CHANNEL ESTIMATION BASED ON GRADIENT OPTIMIZATION TECHNIQUES**
- (75) Inventors: Timothy A. Thomas, Palatine, IL (US); Frederick W. Vook, Schaumburg, IL (US)
- (73) Assignee: Motorola, Inc., Schaumburg, IL (US)
- (*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(h) by 608 days.
- (21) Appl. No.: 09/608,582
- (22) Filed: Jun. 30, 2000
- (51) Int. Cl.⁷ H04B 1/10
- (52) U.S. Cl. 375/349; 375/346; 375/285; 455/63; 455/278.1
- (58) Field of Search 375/316, 230-234, 375/235, 262, 265, 267, 341, 347, 346, 144, 148, 349, 285; 455/63, 65, 137, 273, 276.1, 278.1; 342/380; 370/204, 206, 208, 210

Article entitled "Basis Expansions Models and Diversity Techniques for Blind Identification and Equalization of Time-Varying Channels," (Author Georgios B. Giannakis, Proc. IEEE, vol. 86, No. 10, pp. 1969-1986, Oct. 1998).

Article entitled "Characterization of Fast Fading Vector Channels for Multi-Antenna Communication Systems," (Authors Gregory Raleigh, Suhas N. Diggavi, Ayman F. Naguib, Arogyaswami Paulraj, Proc. 28th, Asilomar Conf., Pacific Grove Ca, 5 pp., Nov. 1994).

Article entitled "Deterministic Approaches for Blind Equalization of Time-Varying Channels with Antenna Arrays," (Authors Hui Liu, Georgios B. Giannakis, IEEE Trans on Sig. Proc., vol. 46, No. 11, pp. 3003-3013, Nov. 1998).

Article entitled "Least-Squares Multi-User Frequency-Domain Channel Estimation for Broadband Wireless Communication Systems," (Authors Timothy A. Thomas, Fred W. Vook and Kevin L. Baum, 37th Allerton Conference, Monticello, IL, 10 pp., Sep. 1999).

Article entitled "Linear and Nonlinear Programming," (Author David G. Luenberger, Addison-Wesley Publishing Company, Menlo Park, CA, pp. 215-216, 1989).

Article entitled "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come," (Author John A. C. Bingham, IEEE Comm. Mag., vol. 28, pp. 5-14, May 1990).

(56) **References Cited**

(List continued on next page.)

U.S. PATENT DOCUMENTS

- 5,906,721 A * 5/1999 Liu et al. 370/342
- 5,912,876 A * 6/1999 H'miny 370/210
- 6,104,000 A * 8/2000 Raleigh et al. 455/561
- 6,141,913 A * 10/2000 Thomas et al. 375/347
- 6,288,015 H1 * 10/2001 Heiskala 370/210
- 6,442,129 H1 * 8/2002 Yonge et al. 370/204
- 6,473,913 H1 * 10/2002 Ariyavistakul et al. 370/203

OTHER PUBLICATIONS

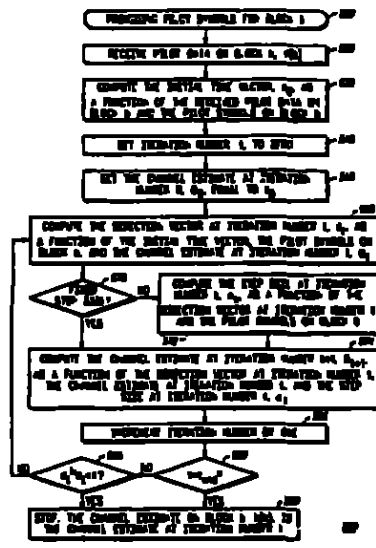
Article entitled "Adaptive Frequency-Domain Equalization and Diversity Combining For Broadband Wireless Communication," (Author, Martin V. Clark, IEEE JSAC, vol. 16, pp. 1385-1395, Oct. 1998).

Primary Examiner—Phuong Phu
(74) Attorney, Agent, or Firm—Kenneth A. Haas

(57) **ABSTRACT**

The invention is a method of estimating the time and frequency response of at least one desired signal received by at least one antenna, based on Gradient Optimization techniques. Through the use of gradient optimization techniques and efficient DFT processing, the invention provides frequency response estimates that require fewer computations than prior art decision-directed channel estimation devices.

21 Claims, 7 Drawing Sheets



OTHER PUBLICATIONS

Article entitled "*Robust Channel Estimation for OFDM Systems with Rapid Dispersive Fading Channels*," (Authors Ye G. Li, Leonard J. Cimini, Jr., Nelson R. Sollenberger, *IEEE Trans. On Comm.*, vol. 46, pp. 902-915, Jul. 1998).

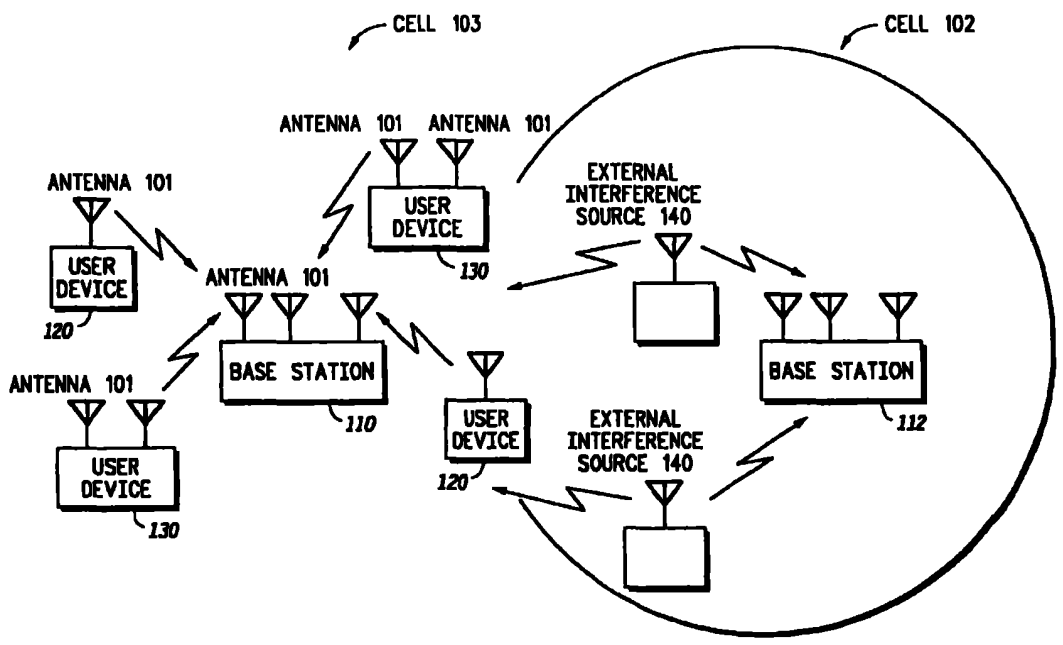
Article entitled "*Sinusoidal Model and Prediction of Fast Fading Processes*," (Authors Jeng-Kuang Hwang, Jack H. Winters, *Globecom*, pp. 892-897, 1998).

Article entitled "*Space-Time Modems for Wireless Personal Communications*," (Author A. J. Paulraj, Boon C. Ng, *IEEE Personal Communications Magazine*, pp. 36-48, Feb. 1998).

Article entitled "*Analysis of DFT-Based Channel Estimators for OFDM*" (Authors Ove Edfors, Magnus Sandell, Jan-Jaap van de Beek, Sara Kate Wilson, Per Ola Börjesson, This work has been presented in part at the 1995 Vehicular Technology Conference (VTC '96) in Chicago, Illinois, Jul. 25-28, 1995, pp. 815-819).

Article entitled "*Channel Estimation for OFDM Systems with Transmitter Diversity in Mobile Wireless Channels*" (Authors Ye(Geoffrey) Li, *Senior Member, IEEE*, Nambirajan Seshadri, *Senior Member, IEEE*, and Sirikiat Ariyavistakul, *Senior Member, IEEE*, pp. 461-471, *IEEE Journal of Selected Areas in Communications*, vol. 17, No. 3, Mar. 1999).

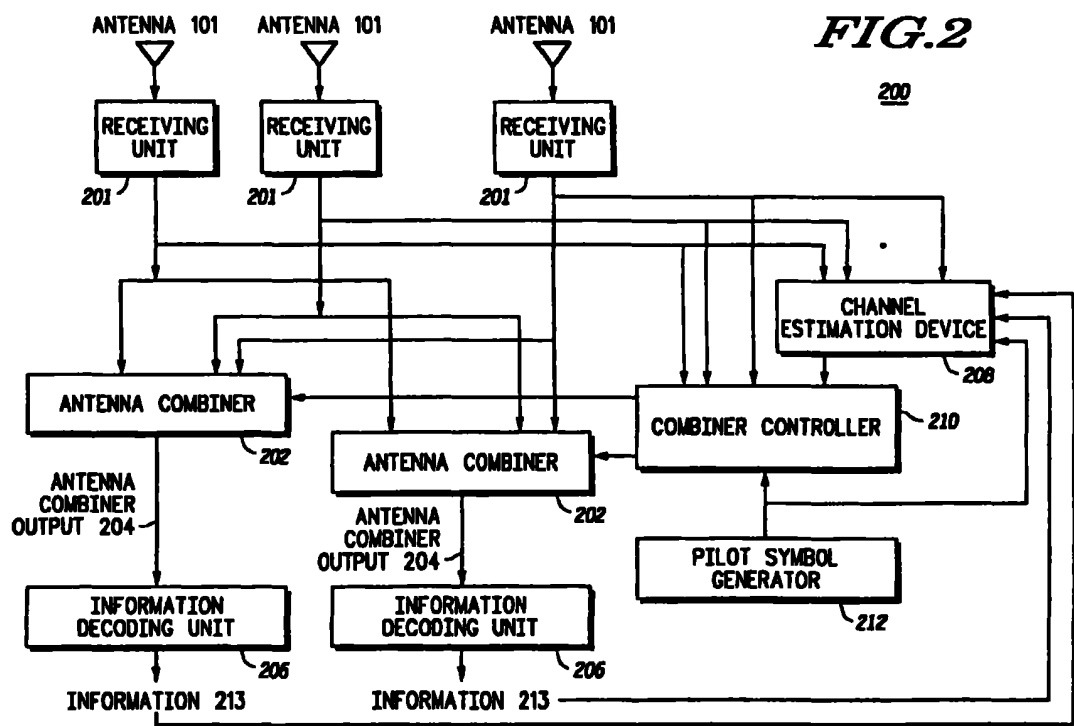
* cited by examiner



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FIG.1

BNSCCD US 6647078 B1

119



120

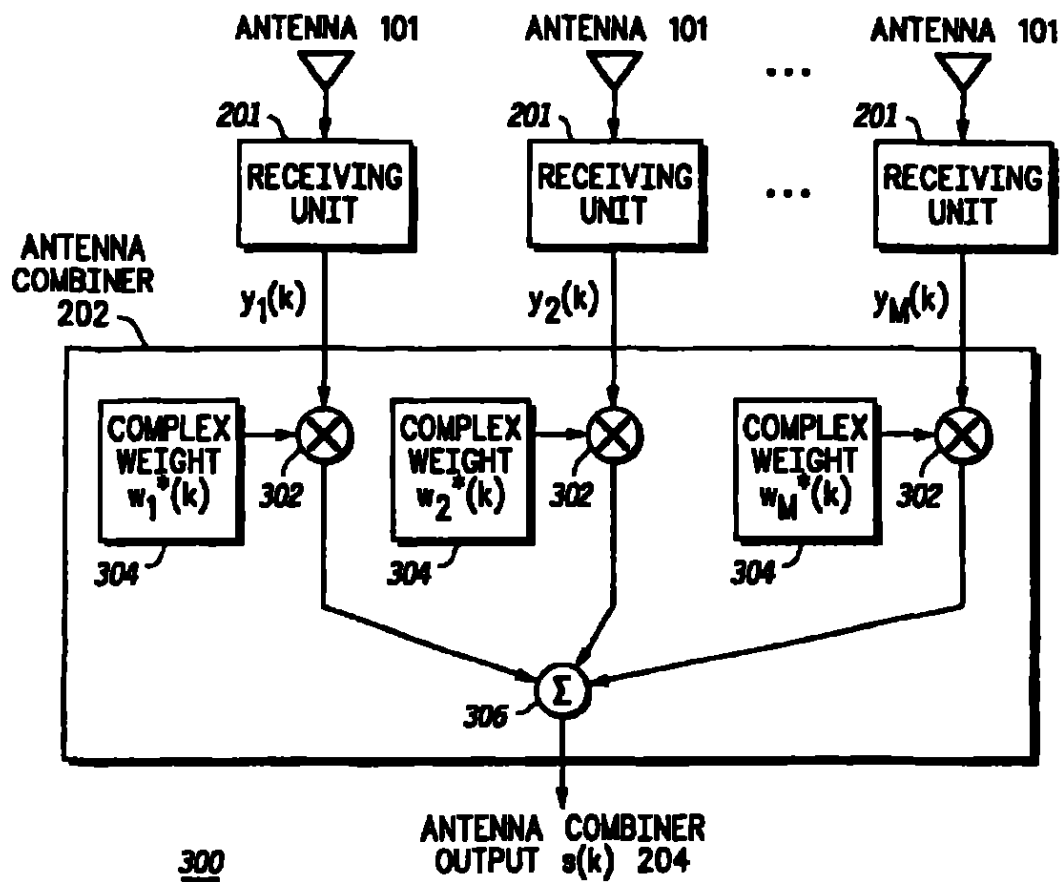


FIG. 3

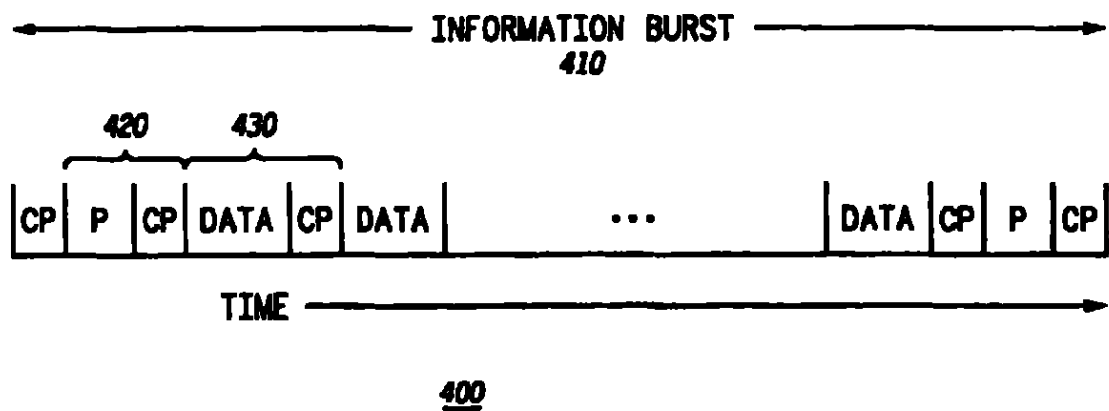


FIG. 4

116

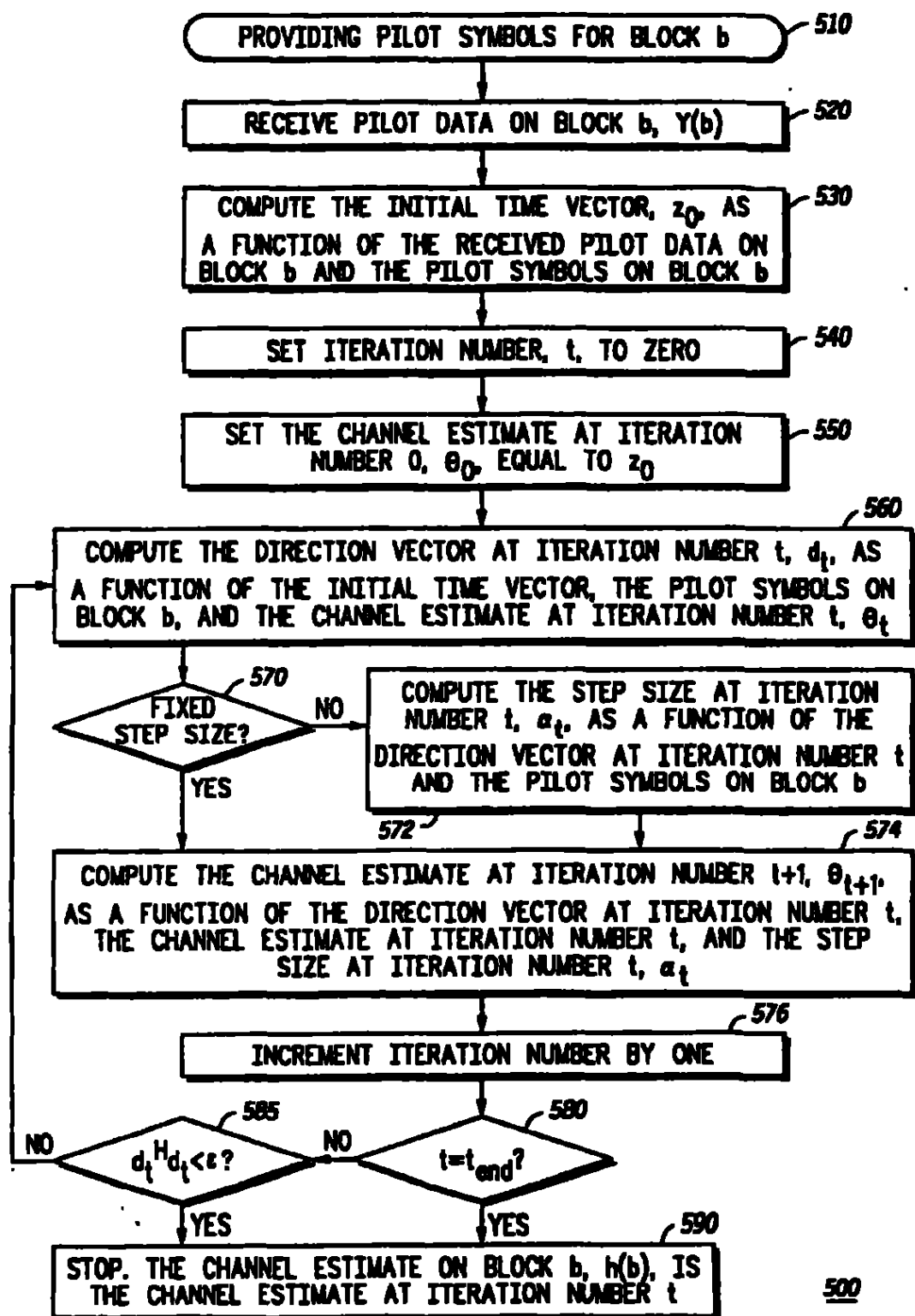


FIG. 5

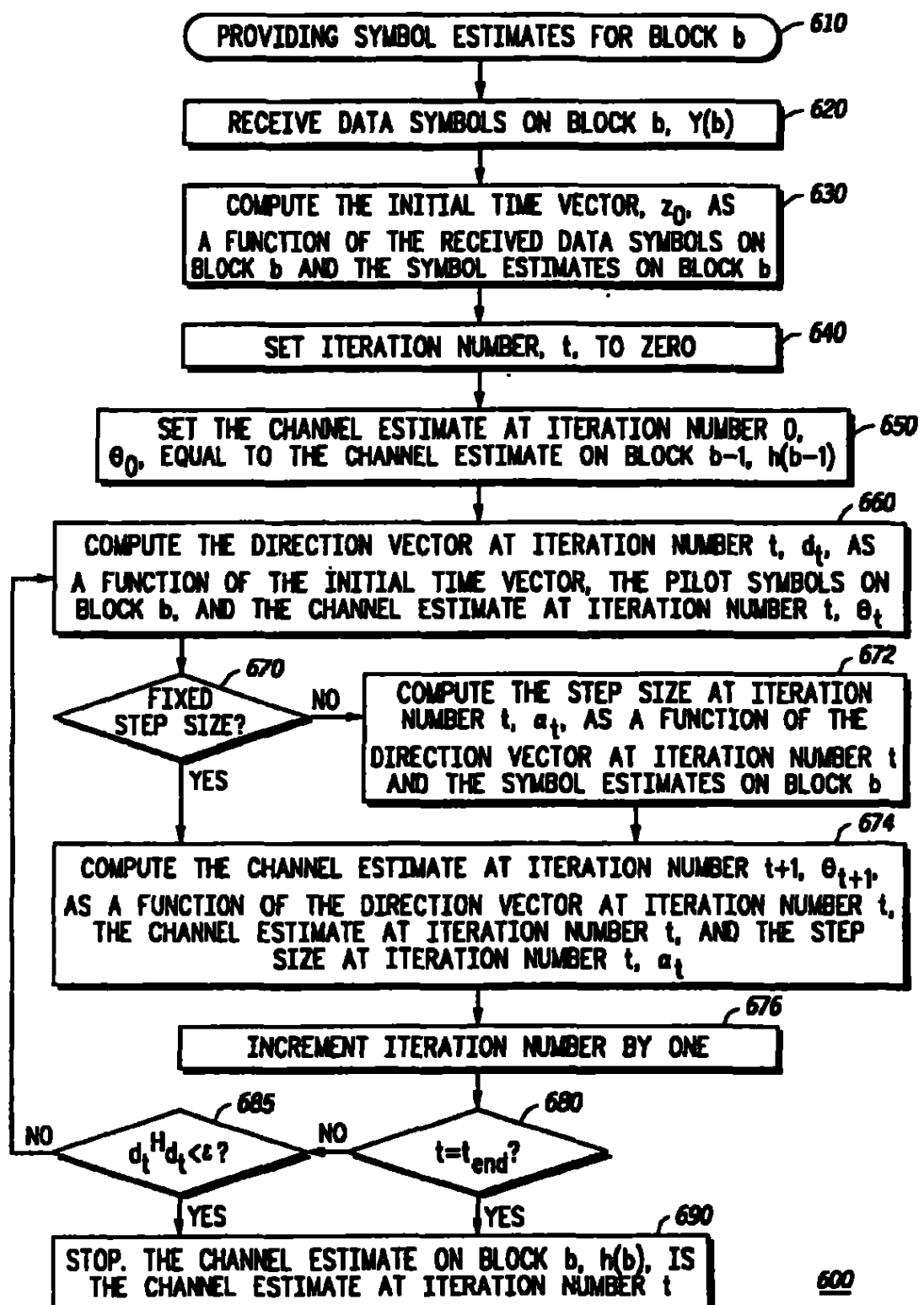


FIG. 6

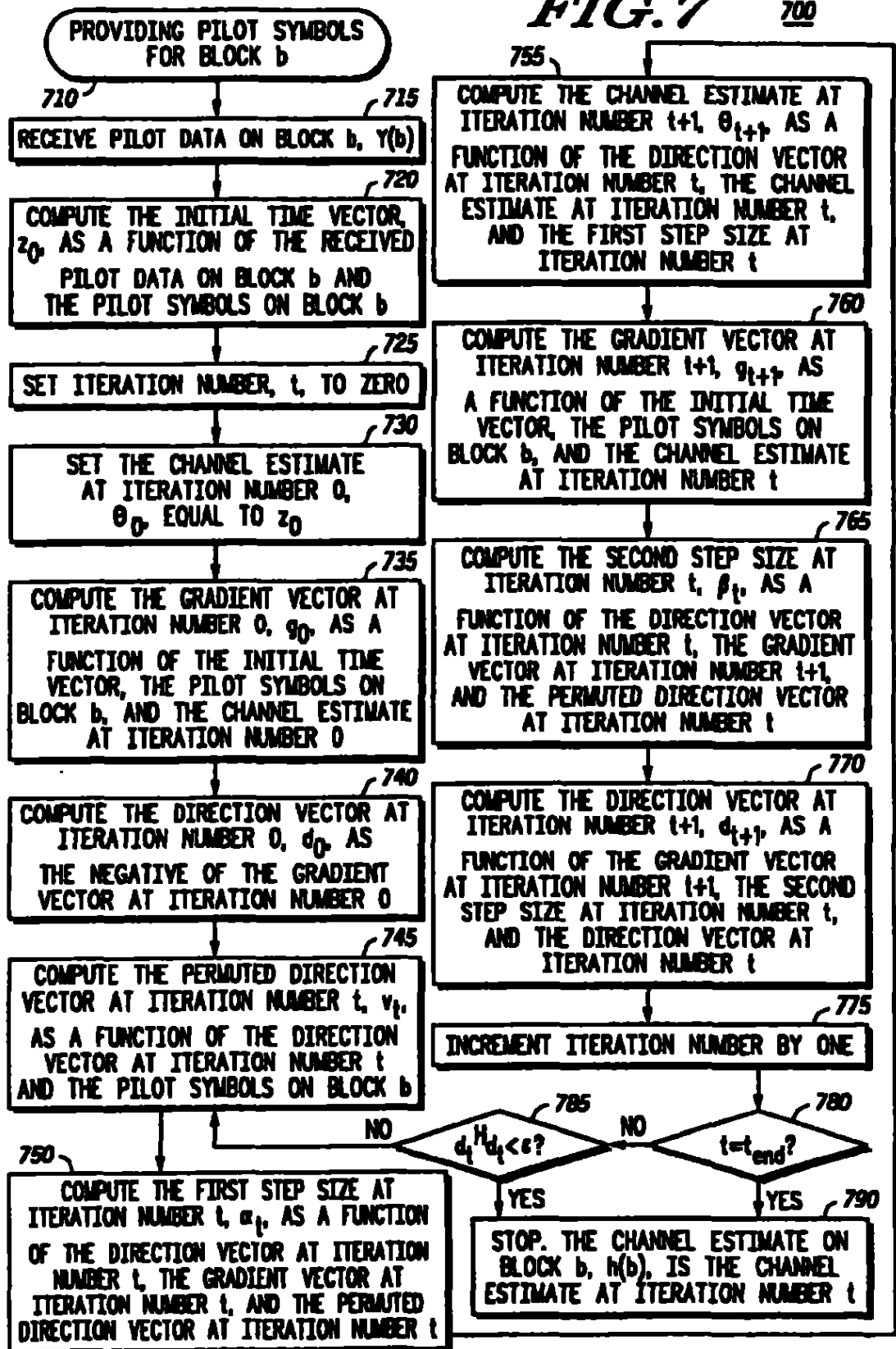
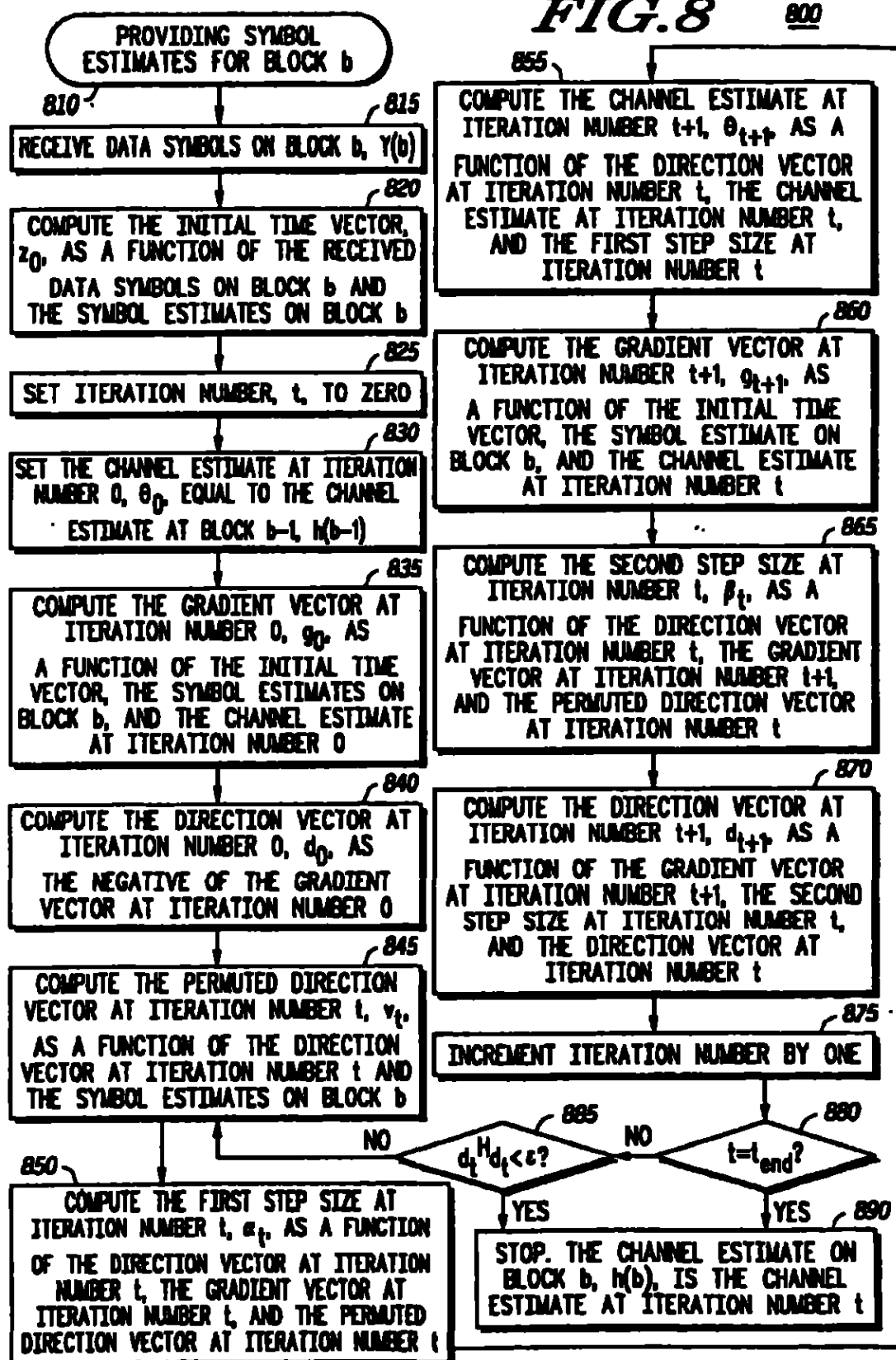
FIG. 7 700

FIG. 8 800

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METHOD AND DEVICE FOR MULTI-USER FREQUENCY-DOMAIN CHANNEL ESTIMATION BASED ON GRADIENT OPTIMIZATION TECHNIQUES

FIELD OF THE INVENTION

The present invention generally relates to the field of communication systems and more particularly, to estimating the time and frequency response of at least one desired signal received by at least one antenna, based on Gradient Optimization techniques.

BACKGROUND OF THE INVENTION

In a wireless communication system, a major design challenge is to maximize system capacity and performance in the presence of interference, and a time-varying multipath channel. Multipath propagation is caused by the transmitted signal reflecting off objects near the transmitter and receiver and arriving at the receiver over multiple paths. Interference in a communication system can come from a variety of sources depending on the particular system deployment. If the system is in motion, then Doppler-induced channel variations become an issue. Interference and multipath are major factors that limit the achievable performance and capacity of a communication system because both effects interfere with the ability of a communication receiver to properly decode the transmitted data.

In a multipath propagation channel, the transmitted signal propagates to the receiver over a finite number L_p of propagation paths, where each path has an associated time delay and complex gain. In such a channel, the communication receiver receives the superposition of L_p delayed, attenuated, and phase-shifted copies of the transmitted signal. The number of paths L_p and their time delays and phase shifts depends on the physical location of the various scattering objects (such as buildings, automobiles, and trees) in the immediate vicinity of the transmitter and receiver. The complex attenuation (magnitude and phase) of each path depends on the length of each path, as well as the material composition of any scatterers or reflectors encountered along the path.

The presence of multipath can severely distort the received signal. In a multipath environment, the multiple copies of the transmitted signal can interfere constructively in some portions of the occupied bandwidth. In other portions of the occupied bandwidth, the multiple copies can interfere destructively at the receiver. This signal duplication causes unwanted variations in the received signal strength over the bandwidth occupied by the signal. Furthermore, if the difference in the path delays of the various propagation paths is significantly greater than the duration of a transmitted information symbol, then intersymbol interference is present at the receiver. When intersymbol interference is present, the received signal is corrupted by prior transmitted symbols propagating over paths having delays relative to the shortest path that are longer than the duration of an information symbol. The demodulation process (the process of determining which information symbol was transmitted) becomes difficult in the presence of intersymbol interference.

In a mobile wireless communication system, the complex attenuation of each of the multipath components of the received signal becomes a time-varying function of the transmitter's path and speed throughout the scattering field local to the transmitter's position. The transmitter's motion

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causes the received signal strength at a particular portion of the occupied bandwidth to vary as time progresses. In a mobile multipath channel, the overall channel response not only varies across the occupied bandwidth of the signal, but also across time as well.

In addition to multipath, interference is another system component that limits the performance of a communication system. If the system is deployed in an unlicensed band, then other users of the band can generate interference. And in a cellular system employing frequency reuse, transmitters in another cell that is allocated the same set of frequency channels can generate co-channel interference. Frequency reuse is the practice of assigning the same frequency channels to multiple users of the allocated spectrum.

Many cellular communication systems employ the technique of frequency reuse in order to maximize the utilization of the frequency spectrum allocated to a wide-area system deployment. In a cellular system, a large geographical area is divided into smaller regions called cells, where each cell is served by a single base station operating on an assigned set of frequency channels. Within each cell, multiple subscriber devices are allowed to communicate with the base station on the frequency channels assigned to that cell. The concept of frequency reuse involves allocating different sets of frequency channels to the cells belonging to a particular group and then reusing the same sets of frequencies to the cells belonging to another group of cells.

The reuse factor of a cellular system is defined to be the minimum distance between two cells that are allocated the same set of frequency channels divided by the radius of a cell. A cellular system employing a large reuse factor does not utilize the allocated spectrum as efficiently as a cellular system employing a smaller reuse factor. However, the level of co-channel interference received by a receiver in the cellular system is directly dependent on the reuse factor. Reducing the reuse factor tends to increase the level of co-channel interference experienced by a receiver. To better utilize the available spectrum, it would be advantageous to be able to suppress the effects of co-channel interference.

To compensate for co-channel interference, multi-user channel estimation is used. The current multi-user channel estimators require a very large matrix inversion which is based on each user's pilot sequence or symbol decisions to be solved to find a matrix of filtering coefficients that, when multiplied by the received data, gives the channel estimates. This requires processing and storing a large matrix that can be burdensome to processor and memory limited implementations. Also, there is a need to perform a large matrix inversion based on each user's pilot sequences or symbol decisions. The inverse is not a problem when the sequences are known since this inverse is computed once upon device initialization and then stored. However, the large inverse makes decision-directed channel updates impractical in real time.

Thus, there is a need for a method and device for estimating the time and frequency response of at least one transmitted signal received on at least one receive antenna. In addition, there is a need for a method and device to eliminate the need for computing and storing the inverse of a very large matrix.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is an overview diagram of a preferred embodiment of a cellular communication system in accordance with the present invention;

FIG. 2 is a block diagram illustrating a preferred embodiment of a device in accordance with the present invention;

FIG. 3 is a block diagram illustrating details of the Antenna Combiner of the device of FIG. 2;

FIG. 4 is a representation of an information burst for Single or Multi-carrier Modulation, which can be used by the communication system in which the device in FIG. 2 operates;

FIG. 5 is a flow chart representation of one embodiment of the method performed by the Channel Estimation Device (208) to provide channel estimation for pilot blocks between a transmitting device and a receiving antenna in accordance with the present invention;

FIG. 6 is a flow chart representation of one embodiment of the method performed by the Channel Estimation Device (208) to provide channel estimation for data blocks between a transmitting device and a receiving antenna in accordance with the present invention;

FIG. 7 is a flow chart representation of one embodiment of the method performed by the Channel Estimation Device (208) to provide channel estimation for pilot blocks between a transmitting device and a receiving antenna in accordance with the present invention; and

FIG. 8 is a flow chart representation of one embodiment of the method performed by the Channel Estimation Device (208) to provide channel estimation for data blocks between a transmitting device and a receiving antenna in accordance with the present invention.

DETAILED DESCRIPTION OF THE PRESENTLY PREFERRED EMBODIMENT

An adaptive antenna array is an array of antennas connected to a communications receiver and operates by combining the signals received by the antennas so as to optimize in an adaptive fashion the receive characteristics of the array. By weighting and then summing the multiple antenna signals, the adaptive antenna array can adapt its angular response, sometimes called the array pattern, while it operates in response to changes in the propagation environment. While operating, the adaptive antenna attempts to maximize the reception of the signal received from a desired transmitting device, as it simultaneously minimizes the effects of all other interfering signals and noise. In a communication system, the interference suppression capability of an adaptive antenna array offers the potential to reduce co-channel interference, improve coverage quality, and increase overall system capacity.

Adaptive antenna arrays also offer the possibility of providing a new way of multiplexing multiple devices: Spatial Division Multiple Access (SDMA). With SDMA, multiple devices can simultaneously share the same channel (i.e., time, frequency, or code channel) and are separated and simultaneously decoded by the receiver array on the basis of their spatial position relative to the antenna array. When successfully deployed, SDMA promises to provide enormous increases in system capacity.

Spatial Division Multiple Access is a difficult technology to implement because of the rapidly varying multipath fading channel. In an SDMA system, the devices that are sharing a channel provide interference to the receiver processing algorithms that must decode the signals transmitted by the devices. When a receiver-processing algorithm attempts to decode one SDMA device, the other SDMA devices provide strong interference to the decoding algorithm. The adaptive antenna implementing SDMA suppresses the effects of the other devices when attempting to decode one of the transmitting devices.

A problem associated with employing both an equalizer and an adaptive antenna in a wireless communication system

lies in the design of an algorithm and device having adequate ability to adapt to changes in the signal environment. For best performance, adaptive antennas that operate in a fast-fading multipath environment must adapt to the rapidly varying channel as well as to any changes in the nature of the desired and interfering signals. In a broadband system, a frequency-selective multipath channel will cause significant variations in the channel across the occupied bandwidth of the received signal. Equalization and interference-suppression algorithms, which cannot track these channel variations both in time and frequency, will suffer significant degradation in performance as measured by the Bit-Error Rate (BER) or Signal-to-Interference-plus-Noise Ratio (SINR).

A common ingredient in many equalizers and adaptive antenna arrays is an algorithm and device that estimates the characteristics of the multipath propagation environment, channel transfer function, or channel frequency response, between the desired transmitting device and each of the at least one receiving antenna at the communication receiver. When SDMA is being employed in a broadband system operating in a frequency-selective environment, then an adaptive antenna array generally requires an estimate of the channel frequency response between each receiving antenna and each of the transmitting devices that are simultaneously sending information to the array. The channel estimation algorithm in this case should operate to simultaneously solve for the channel responses of the multiple transmitting devices.

Given the need for higher system capacities in wireless communication systems, adaptive antennas and advanced equalization techniques are important for satisfying the ever-increasing need for high data rate communications. To support the operation of adaptive antennas and equalizers, it is advantageous to have a method and device that tracks the time-varying frequency response of a broadband system accurately enough to permit effective equalization and interference suppression. Given the difficulties associated with implementing SDMA and the deleterious nature of the multipath-fading environment, such a device would enable improved equalization and interference suppression performance in a mobile broadband communication system.

Briefly described, the present invention is a method and device for calculating the channel frequency responses of the signals transmitted by at least one transmitting device and received on at least one receive antenna. The invention utilizes gradient optimization techniques and efficient Discrete Fourier Transform (DFT) processing that significantly lowers channel estimation complexity with no loss in channel estimation performance. Using this method eliminates the need to compute the inverse of a very large matrix that is required by most multi-user channel estimators. Also, this invention eliminates problems associated with channel estimations at edge subcarriers in OFDM (Orthogonal Frequency Division Multiplexing), thus providing improved symbol estimation performance.

The method and device of the present invention can be incorporated into a communications receiving device, base station, or subscriber unit. In the present invention, the term "device" can refer to any type of communications device such as a base station, subscriber unit, or other communications receiver or transmitter.

The present invention is implemented in a communication system where at least one desired transmitting device transmits information to a communication receiver having at least one antenna. In a preferred embodiment, pluralities of

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transmitting devices simultaneously transmit information to a communication receiver having a plurality of antennas. A transmitting device transmits its information in bursts that contain two components: a training interval and a data interval, also called blocks. The information transmitted in a training interval contains pilot symbol sequences of content and duration known by both the transmitting device and the communication receiver. The data interval contains data symbols that must be recovered at the receiving device. In the present invention, the term "burst" refers to any one of the following: a short or isolated transmission, a portion of a longer transmission, a portion of a continuous transmission, a portion of a semi-continuous transmission, a time-limited transmission, a bandwidth-limited transmission, or any combination thereof.

The ability to receive and recover the information transmitted simultaneously by more than one device significantly increases the capacity of the communication system. Conventional cellular systems permit only one device to transmit on a particular frequency channel within a cell for any predetermined time interval. Providing for more than one device to transmit to a base station on the same frequency channel at the same time will multiply the capacity of the system by a factor equal to the number of devices that are allowed to simultaneously transmit. Devices incorporating algorithms for providing this capability must have the ability to track any variations in the time and frequency-selective channel. The method and device of the present invention provide time-varying channel estimates for use in adaptive antenna equalization estimation techniques, making communication systems more efficient.

A preferred embodiment of the present invention described below typically operates in a time-varying delay-spread channel and operates under the assumption that the channel can change significantly over the occupied bandwidth and significantly in time. This requires a frequency-domain multi-user channel estimation technique for tracking the time and frequency variations of multiple SDMA users sharing the same time frequency channel.

FIG. 1, numeral 100, illustrates a wireless communication system in accordance with the preferred embodiment of the present invention. As shown in FIG. 1, a Base Station 110 provides communication service to a geographic region known as a cell 103. At least one User Devices 120 and 130 communicate with the Base Station 110. In some embodiments of the communication system of FIG. 1, at least zero External Interference Sources 140 share the same spectrum allocated to the base station 110 and subscriber devices 120 and 130. The External Interference Sources 140 represent an unwanted source of emissions that interferes with the communication process between the Base Station 110 and the User Devices 120 and 130. The exact nature and number of the External Interference Sources 140 will depend on the specific embodiment of the communication system of FIG. 1. In some cases, as is shown in FIG. 1, an External Interference Source will be another User Device 140 (similar in construction and purpose to User Device 120) that is communicating with another Base Station 112 in the same frequency spectrum allocated to Base Station 110 and User Devices 120 and 130. As shown in FIG. 1, User Devices 120 has a single antenna, while User Devices 130 have at least one antenna. The method and device of the present invention can be implemented as part of a Base Station 110 as well as part of a User Device 120 or 130.

FIG. 2, numeral 200, is a block diagram illustrating a device in accordance with the present invention. The communication receiver in accordance with the present inven-

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tion includes at least one antenna (101) wherein the outputs of the antennas are each provided to a receiving unit (201). The outputs of the receiving units (201) are provided to at least one Antenna Combiner (202). The signals from the receiving units (201) are also fed into the Combiner Controller (210), which regulates the operation of the at least one Antenna Combiner (202). The signals from the receiving units (201) are also fed into the Channel Estimation Device (208). The Pilot Symbol Generator (212) generates pilot symbol information that is used by the Combiner Controller (210) to control the Antenna Combiner (202). The pilot symbol information generated by the Pilot Symbol Generator (212) is also used by the Channel Estimation Device (208) to estimate the time-varying frequency responses of the transmitting devices (110, 112, 120, 130, or 140, or any combination thereof). The output of an Antenna Combiner (202) is fed into an Information Decoding Unit (206), which decodes the Antenna Combiner Output (204) and generates data information (213) that was received by the Antennas (101). The output of the information decoding unit (206) is fed into the channel estimation device (208) to estimate the time-varying frequency response of the transmitting devices.

FIG. 3, numeral 300, is a block diagram illustrating details of the Antenna Combiner of the device of FIG. 2. Antenna Combiner (202) is coupled to the receiving units (201), which in turn are coupled to the antennas (101). In a preferred embodiment, the receiving units (201) may include radio frequency pre-amplifiers, filters, and other devices that can be used to convert the radio frequency signal received by the antenna down to a digital stream of base b and equivalent complex symbols. As shown in FIG. 2, the output of the i 'th receiving unit (201) (where i is an integer between 1 and M inclusive, and M is the total number of antenna elements) is mathematically denoted by $y_i(k)$, where k and i are integers, and is provided to the antenna combiner (202) which can be in the form of a plurality of complex multipliers (302) which multiply the output of each receiving unit (201) by a complex weight (304), mathematically denoted as $w_i(k)$, and a combiner (306) sums the outputs of the plurality of complex multipliers (302). The values of the complex weights (304) are controlled by the Combiner Controller (210), shown in FIG. 2.

FIG. 4, numeral 400, is a timing diagram illustrating the structure of an information burst for single or multi-carrier modulation (410) transmitted between a plurality of transmitting devices (e.g., Base station 110, User Device 120, or 130) and a receiving device (e.g., Base station 110, User Device 120, or 130). An information burst (410) includes at least one training interval (420) also referred to as a pilot block, and at least one data interval (430) also referred to as a data block. During a training interval (420), a transmitting device (e.g., a Base station 110, or a User Device 120, or a User Device 130, or a base station 112, or in some cases a User Device 140, or any combination thereof) transmit training information consisting of at least one pilot symbol of duration, content, and timing known to both the transmitting device and the receiving device (a Base station 110, or a User Device 120, or a User Device 130). The Channel Estimation Device (208) in the present invention uses the outputs of the receiving units (201) during at least one of the training intervals (420) and the knowledge of the transmitted sequence of pilot symbols to compute an estimate of the channel time and frequency response between the plurality of transmitting devices (a Base station 110 or 112, or a User Device 120, 130, or 140, or any combination thereof) and the antennas (101) of the receiving device. To update the

channel estimate, the channel estimation device (208) in the present invention uses the outputs of the information decoding units (206) and the outputs of the receiving units (201) during at least one data interval (430) to compute an estimate of the channel time and frequency response between the plurality of transmitting devices (a Base station 110 or 112, or a User Device 120, 130, or 140, or any combination thereof) and the antennas (101) of the receiving device. This information burst acts in accordance with all channel estimations using cyclic prefixes.

The channel estimation device for the present invention utilizes the mathematical architecture of gradient optimization. Gradient optimization is a finite iterative procedure for solving a system of n equations in n unknowns. A description of a frequency-domain channel estimation based on gradient optimization techniques is now given:

For the derivation of the gradient methods, assume that the entire block of symbols (e.g., 420 or 430 in FIG. 4) to be processed (in OFDM this block is sometimes referred to as a baud or as an OFDM symbol interval) is of known symbols (either pilots, if the block is 420, or symbol decisions, if the block is 430). The approach to be described can be extended to arbitrary pilot formats as well. Also the following derivation will be given for one of the at least one receiving antennas. An OFDM (Orthogonal Frequency Division Multiplexing) system is assumed, although the derivation is applicable to any system employing frequency-domain processing. The OFDM system described below employs an FFT size of N , and K frequency-domain subcarriers are employed to transmit information (either pilot or data). Also, in the following derivation, the term "user" is used to refer to a transmitting antenna so that the derivation will be applicable to conventional single user systems, SDMA (Spatial Division Multiple Access) systems, MIMO (Multiple-Input-Multiple-Output) systems, space-time-coding systems, or any other related system having at least one transmitting antennas and at least one receiving antenna. Additionally in the following derivations, the symbol decisions do not necessarily have to be hard sliced to actual symbol values. Some type of soft decision can be used such as hyperbolic tangent mappings.

The received data on one of the receive antennas, ignoring noise and unknown interference, is given as:

$$Y(k, b) = \sum_{j=1}^J H_j(k, b) x_j(k, b) \quad (1)$$

where k is the subcarrier number (k ranges from 1 to K), b is a number identifying the baud being processed (b ranges from 1 to B), J is the number of Spatial Division Multiple Access (SDMA) users (the number of devices or antennas transmitting simultaneously), $H_j(k, b)$ is user j 's frequency-domain channel at subcarrier k , baud b , and $x_j(k, b)$ is user j 's frequency-domain symbol on subcarrier k , baud b (in single carrier, $x_j(k, b)$ is the DFT of user j 's K time-domain symbols). In a preferred environment, the Discrete Fourier Transform (DFT's) are replaced by Fast Fourier Transform (FFT's) to simplify calculations. $H_j(k, b)$ is mathematically modeled as the N -point DFT ($N \geq K$) of a length L sequence, $h_j(l, b)$ (where $l=0, 1, \dots, L-1$ or any other appropriate length L integer sequence), of time-domain tap gains (called time-taps) as follows: (Note that the length L sequence $h_j(l, b)$ can vary as a function of the baud index b .)

$$H_j(k, b) = \sum_{l=0}^{L-1} h_j(l, b) e^{-j2\pi k l / N} \quad (2)$$

In single carrier employing frequency-domain receive processing $K=N$, but in OFDM, $N > K$ so that the estimator assumes a low correlation between the channel at subcarrier $k=1$ and the channel at subcarrier $k=K$.

In matrix form and extended to all K subcarriers, the previous equation becomes:

$$(K \times 1) H_j(b) = \begin{bmatrix} H_j(1, b) \\ \vdots \\ H_j(K, b) \end{bmatrix} = F H_j(b) \quad (3)$$

where:

$$(K \times L) F = \begin{bmatrix} f^H(0) \\ \vdots \\ f^H(K-1) \end{bmatrix}, \quad (L \times 1) f(k) = \begin{bmatrix} 1 \\ e^{-j2\pi k / N} \\ \vdots \\ e^{-j2\pi k(L-1) / N} \end{bmatrix} \quad (4)$$

$$(L \times 1) H_j(b) = \begin{bmatrix} h_j(0, b) \\ \vdots \\ h_j(L-1, b) \end{bmatrix}$$

The received data on baud b at each subcarrier, stacked into a vector, $Y(b)$, can be written as:

$$(K \times 1) Y(b) = \begin{bmatrix} Y(1, b) \\ \vdots \\ Y(K, b) \end{bmatrix} = \sum_{j=1}^J X_j(b) F H_j(b) = X(b) D H(b), \quad (5)$$

where $(K \times K) X_j(b) = \text{diag}(x_j(1, b), \dots, x_j(K, b))$, (diag means the matrix is composed of all zeros except for the diagonal elements which are given in parenthesis) and

$$(K \times K) X(b) = [X_1(b) \dots X_J(b)], \quad (6)$$

$$(J K \times J L) D = \begin{bmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & F \end{bmatrix}, \quad (J L \times 1) H(b) = \begin{bmatrix} H_1(b) \\ \vdots \\ H_J(b) \end{bmatrix}$$

The least squares estimator is the solution to the following minimization problem:

$$\min_{H(b)} \|X(b) D H(b) - Y(b)\|^2 = \min_{H(b)} H^H(b) D^H X^H(b) X(b) D H(b) - \quad (7)$$

$$H^H(b) D^H X^H(b) Y(b) - Y^H(b) X(b) D H(b) + Y^H(b) Y(b)$$

The gradient of the previous equation with respect to $H(b)$ is given as (where a multiplicative factor of 2 is neglected to improve the clarity of the presentation):

$$\Delta H(b) = D^H X^H(b) X(b) D H(b) - D^H X^H(b) Y(b) \quad (8)$$

where H is the conjugate transpose. When computing this gradient, note that a matrix multiplication by the large matrix D can be implemented as J N -point DFT's with each

DFT truncated to the first K samples. Also, multiplication by D^H can be implemented as J N-point inverse Discrete Fourier transform (IDFTs) with each IDFT truncated to the first L samples. Note that $X(b)$ is a sparse matrix, so only JK complex multiplies are required when it is multiplied by a JKx1 vector. In other words, in computing the above gradient, $Dh(h)$ can be found by taking a separate FFT of each user's L time taps in $h(b)$, truncating each to K samples, and then stacking the result into a JKx1 vector. Then $X(b)(Dh(h))$ can be found by JK complex multiplies (and some addition operations). Next $X^H(b)(X(b)(Dh(h)))$ can be found by JK complex multiplies, and finally, $D^H(X^H(b)(X(b)(Dh(h))))$ can be found through JN point IDFTs truncated to L samples and stacked into a JLx1 vector. Calculating the gradient with these methods (that is, with the DFT or FFT operations) requires significantly less operations than if matrix multiplication operations are carried out directly as written in the Equation (8)). It is these observations that lead to a low computational complexity decision-directed algorithm based on the gradient methods of optimization. These processes are described in the following two lists of steps (the first for the gradient method, the second for the conjugate gradient method) but are explained in detail with FIG. 5, through FIG. 8.

Fixed Step Size and Steepest Descent Gradient Method

- 1. $z_0 = D^H X^H(b) Y(b)$, $t=0$,
 $\theta_0 = z_0$ unless in decision-directed mode where $\theta_0 = h(b-1)$
- 2. $d_t = -D^H X^H(b) X(b) D \theta_t$, $\theta_{t+1} = \theta_t + \alpha_t d_t$,
where $\alpha_t = 1$ for fixed step size gradient and for steepest descent it equals:

$$\alpha_t = \frac{d_t^H d_t}{d_t^H d_t}, \quad v_t = X(b) D d_t$$

- 3. $t=t+1$. If $t=t_{max}$ then stop and $h(b) = \theta_t$, else go to 2

Conjugate Gradient Method

- 1. $z_0 = D^H X^H(b) Y(b)$, $t=0$,
 $\theta_0 = z_0$ unless in decision-directed mode where $\theta_0 = h(b-1)$
- 2. $g_0 = D^H X^H(b) X(b) D \theta_0 - z_0$, $d_0 = -g_0$

$$3. \alpha_t = -\frac{g_t^H g_t}{g_t^H X(b) D d_t}, \quad v_t = D^H X^H(b) X(b) D d_t$$

- 4. $\theta_{t+1} = \theta_t + \alpha_t d_t$
- 5. $g_{t+1} = D^H X^H(b) X(b) D \theta_{t+1} - z_0$

$$6. \beta_t = \frac{g_{t+1}^H g_t}{g_t^H g_t}$$

- 7. $d_{t+1} = -g_{t+1} + \beta_t d_t$
- 8. $t=t+1$. If $t=t_{max}$ then stop and $h(b) = \theta_t$, else go to 3.

FIG. 5 shows a flow chart that illustrates the steps performed by the Channel Estimation Device (208) when the fixed step size gradient or steepest decent gradient method is used. This is in accordance with the present invention when block b (equation 1) is a pilot block (420). The pilot symbols for block b 510 are provided by the Pilot Symbol Generator 212. After receiving the pilot symbols and the pilot data (520) on block b, their information is stacked into a vector $Y(b)$ and is equated as

$$(K \times 1) Y(b) = \begin{bmatrix} Y(1, b) \\ \vdots \\ Y(K, b) \end{bmatrix}$$

The initial time vector, z_0 , is computed (530) as a function of the received pilot data on block b and the pilot symbols on block b, the pilot block b being the segment 420. This is such that $z_0 = D^H X^H(b) Y(b)$ where:

$$(K \times JK) X(b) = \{X_1(b) \dots X_J(b)\}, (JK \times JL) D = \begin{bmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & F \end{bmatrix}$$

$(K \times K) X_j(b) = \text{diag}(x_j(1, b), \dots, x_j(K, b))$ diag means the matrix is composed of all zeros except for the diagonal elements which are given in parenthesis. $x_j(1, b)$ is user j's pilot symbol on subcarrier k and block b. For the matrix above, F is determined by:

$$(K \times L) F = \begin{bmatrix} f^H(0) \\ \vdots \\ f^H(K-1) \end{bmatrix} \quad \text{where } (L \times 1) f(k) = \begin{bmatrix} 1 \\ \vdots \\ e^{j2\pi k} \end{bmatrix}$$

(In the preferred embodiment, $z_0 = D^H X^H(b) Y(b)$, would be computed by FFTs and an element-wise multiplication of two vectors.) Next (540) the iteration number t, is set to zero. The initialization also requires that the channel estimate at iteration number 0, θ_0 , be equal to an initial channel estimate. In the preferred embodiment, the initial channel estimate is the initial time vector z_0 (550). Next, in block 560, the direction vector d, is computed at iteration number t, forming d, as a function of the initial time vector, the pilot symbols on block b, and the channel estimate at iteration number t, θ_t . The direction vector is used to determine how big a step should be computed for the next iteration of process 560 through 585. The equation for (560) is $d_t = -D^H X^H(b) X(b) D \theta_t$, (570) If the step size is not fixed ($\alpha_t \neq \alpha_0$, where α_0 where is a constant) then (672) the step size is computed at iteration number t, α_t , as a function of the direction vector at iteration number t and the pilot symbols on block b. The equation for 672 is

$$\alpha_t = \frac{d_t^H d_t}{d_t^H d_t}$$

where $v_t = X(b) D d_t$, 570 The channel estimate is computed at iteration number t+1, θ_{t+1} , as a function of the direction vector at iteration number t, the channel estimate at iteration number t, and the step size at iteration number t, α_t , (574). The equation for 574 is $\theta_{t+1} = \theta_t + \alpha_t d_t$. At this point the iteration number is increased by one (576). If the iteration number t equals t_{max} (580), t_{max} being the number where the channel estimator should stop iterating, then the channel estimate on block b, $h(b)$, is the channel estimate at iteration number t (590). t_{max} is predetermined for the receiving device, and is dependent on its processor power and memory limitations. If $d_t^H d_t < \epsilon$ (585), where $d_t^H d_t$ is the magnitude squared of the direction vector, and in the preferred embodiment, ϵ is a small percentage (e.g., 1%) of the theta information ($\theta^H \theta$), the channel estimate on block b, $h(b)$, is

the channel estimate at iteration t (590). Otherwise the process is repeated from block 560 on until the conditions t equals t_{max} or $d_t^H d_t < \epsilon$ are met, at which point the channel estimate on block b , $h(b)$, is the channel estimate at iteration t (590).

FIG. 6 shows a flow chart that illustrates the steps performed by the Channel Estimation Device (208) when the fixed step size gradient or steepest decent gradient method is used. This is in accordance with the present invention when block b (equation 1) is a data block (430). Symbols for block b , $x_j(k,b)$, (610) are provided by the information decoding unit (206) where $x_j(k,b)$ is user j 's symbol. After receiving the data (620) on block b , it is stacked into a vector and equated as

$$(K \times 1) Y(b) = \begin{bmatrix} Y(1,b) \\ \vdots \\ Y(K,b) \end{bmatrix}$$

The initial time vector, z_0 , is computed (630) as a function of the received data on block b and the symbol estimates on block b . This is such that $z_0 = D^H X^H(b) Y(b)$ where:

$$(K \times JK) X(b) = [X_1(b) \dots X_J(b)], (K \times JL) D = \begin{bmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & F \end{bmatrix}$$

$(K \times K) X_j(b) = \text{diag}(x_j(1,b), \dots, x_j(K,b))$, diag means the matrix is composed of all zeros except for the diagonal elements which are given in parenthesis. $x_j(1,b)$ is user j 's symbol estimate on subcarrier k and block b . For the matrix above, F is determined by:

$$(K \times L) F = \begin{bmatrix} f^H(0) \\ \vdots \\ f^H(K-1) \end{bmatrix}, \text{ where } (L \times 1) f(k) = \begin{bmatrix} 1 \\ e^{j2\pi k \nu} \\ \vdots \\ e^{j2\pi(L-1)k \nu} \end{bmatrix}$$

(In the preferred embodiment, $z_0 = D^H X^H(b) Y(b)$, would be computed by FFT's and an element-wise multiplication of two vectors). Next, (640) the iteration number t , is set to zero. The initialization also requires (650) that the channel estimate at iteration number 0, θ_0 , be equal to an initial channel estimate. In the preferred embodiment, the initial channel estimate is the channel estimate on block $b-1$, $h(b-1)$. This is the channel estimate that has been previously computed for the block 590 if block $b-1$ is a pilot block and 690 if block $b-1$ is a data block. In block 660, the direction vector d_t is computed at iteration number t , forming d_t as a function of the initial time vector, the symbols estimates on block b , and the channel estimate at iteration number t , θ_t . The direction vector is used to determine how big a step should be computed for the next iteration of process 660 through 685. The equations for (660) are: $d_t = z_0 - D^H X^H(b) X(b) D \theta$ where $\nu = X(b) D d_t$, (670) If the step size is not fixed ($\alpha_t = \alpha_0$, where α_0 is a constant) then (672) the step size is computed at iteration number t , α_t , as a function of the direction vector at iteration number t and the pilot symbols on block b . The equation for (672) is:

$$\alpha_t = \frac{d_t^H d_t}{1 + d_t^H d_t}$$

670 The channel estimate is computed at iteration number $t+1$, θ_{t+1} , as a function of the direction vector at iteration number t , the channel estimate at iteration number t , and the step size at iteration number t , α_t , (674). The equation for 674 is $\theta_{t+1} = \theta_t + \alpha_t d_t$. At this point the iteration number is increased by one (676). If the iteration number t equals t_{max} (680), t_{max} being the number where the channel estimator should stop iterating, then the channel estimate on block b , $h(b)$, is the channel estimate at iteration number t (690). t_{max} is predetermined for the receiving device, and is dependent on its processor power and memory limitations. If $d_t^H d_t < \epsilon$ (685), where $d_t^H d_t$ is the magnitude squared of the direction vector, and in the preferred embodiment, ϵ is a small percentage (e.g., 1%) of the theta information ($\theta^H \theta$), the channel estimate on block b , $h(b)$, is the channel estimate at iteration t (690). Otherwise the process is repeated from block 660 on until the conditions t equals t_{max} or $d_t^H d_t < \epsilon$ are met, at which point the channel estimate on block b , $h(b)$, is the channel estimate at iteration t (690).

FIG. 7 shows a flow chart that illustrates the steps performed by the Channel Estimation Device (208) when the conjugate gradient method is used. This is in accordance with the present invention when block b (equation 1) is a pilot block (420). The Pilot Symbol Generator 212 provides the pilot symbols for block b (710). After receiving the pilot data (715) on block b , where

$$(K \times 1) Y(b) = \begin{bmatrix} Y(1,b) \\ \vdots \\ Y(K,b) \end{bmatrix}$$

the initial time vector, z_0 , is computed (720) as a function of the received pilot data on block b and the pilot symbols on block b , the pilot block b being the segment 420. This is such that $z_0 = D^H X^H(b) Y(b)$ where:

$$(K \times JK) X(b) = [X_1(b) \dots X_J(b)], (K \times JL) D = \begin{bmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & F \end{bmatrix}$$

$(K \times K) X_j(b) = \text{diag}(x_j(1,b), \dots, x_j(K,b))$ diag means the matrix is composed of all zeros except for the diagonal elements which are given in parenthesis. $x_j(1,b)$ is user j 's pilot symbol or symbol estimate on subcarrier k and block b . For the matrix above, F is determined by:

$$(K \times L) F = \begin{bmatrix} f^H(0) \\ \vdots \\ f^H(K-1) \end{bmatrix}, \text{ where } (L \times 1) f(k) = \begin{bmatrix} 1 \\ e^{j2\pi k \nu} \\ \vdots \\ e^{j2\pi(L-1)k \nu} \end{bmatrix}$$

(In the preferred embodiment, $z_0 = D^H X^H(b) Y(b)$, would be computed by FFT's and an element-wise multiplication of two vectors). Next (725) the iteration number t , is set to zero. The initialization also requires that the channel estimate at iteration number 0, θ_0 , be equal to an initial channel estimate. In the preferred embodiment, the initial channel estimate is z_0 (730). In block 735, the gradient vector g is

computed at iteration number 0, forming g_0 as a function of the initial time vector, the pilot symbols on block b , and the channel estimate at iteration number 0. The equation for 735 is $g_0 = D^H X^H(b) X(b) D \theta_0 - z_{0,b}$. Then the direction vector d is computed at iteration number 0, forming d_0 as the negative of the gradient vector at iteration number 0 (740). The permuted direction vector v is computed at iteration number 1, as a function of the direction vector at iteration number 1 and the pilot symbols on block b forming v_p (745). The full equation for 745 is $v_p = D^H X^H(b) X(b) D d_1$. Utilizing the equation

$$\alpha_1 = -\frac{g_1^H d_1}{d_1^H d_1}$$

the first step size α is computed at iteration number 1, as a function of the direction vector at iteration number 1, the gradient vector at iteration number 1, and the permuted direction vector at iteration number 1 forming α_1 (750). Block 755 computes the channel estimate at iteration number $t+1$, as a function of the direction vector at iteration number t , the channel estimate at iteration number t , and the first step size at iteration number t , forming θ_{t+1} , using the equation $\theta_{t+1} = \theta_t + \alpha_t d_t$. Next the gradient vector g is computed at iteration number $t+1$ as a function of the initial time vector, the pilot symbols on block b , and the channel estimate at iteration number t where $g_{t+1} = D^H X^H(b) X(b) D \theta_{t+1} - z_{t+1,b}$ (760). The second step size β is computed at iteration number t as a function of the direction vector at iteration number t , the gradient vector at iteration number $t+1$, and the permuted direction vector at iteration number t forming β_t (765). This is illustrated as

$$\beta_t = -\frac{g_{t+1}^H d_t}{d_t^H d_t}$$

Block 770 computes the direction vector at iteration number $t+1$, d_{t+1} , as a function of the gradient vector at iteration number $t+1$, the second step size at iteration number t , and the direction vector at iteration number t using the equation $d_{t+1} = -g_{t+1} + \beta_t d_t$. At this point the iteration number t is increased by one (775). If the iteration number t equals t_{max} (780), t_{max} being the number where the channel estimator should stop iterating, the channel estimate on block b , $h(b)$, is the channel estimate at iteration number t (790). t_{max} is predetermined for the receiving device, and is dependent on its processor power and memory limitations. If $d_t^H d_t < \epsilon$ (785), where $d_t^H d_t$ is the magnitude squared of the direction vector and in the preferred embodiment, ϵ is some small percentage (e.g., 1%) of the theta information ($\theta^H \theta$), the channel estimate on block b , $h(b)$, is the channel estimate at iteration number t (790). Otherwise the process from block 745 through 785 is repeated until the conditions t equals t_{max} or $d_t^H d_t < \epsilon$ are met, at which point the channel estimate on block b , $h(b)$, is the channel estimate at iteration t (790).

FIG. 8 shows a flow chart that illustrates the steps performed by the Channel Estimation Device (208) when the conjugate gradient method is used. This is in accordance with the present invention when block b (equation 1) is a data block (810), the data block b being the segment 430. Symbols for block b , $x_j(k,b)$ (810) are provided by the information decoding unit (206) where $x_j(k,b)$ is user j 's symbol. After receiving the pilot data on block b , where

$$(K \times 1) Y(b) = \begin{bmatrix} Y(1,b) \\ \vdots \\ Y(K,b) \end{bmatrix}$$

is true, the initial time vector, $z_{0,b}$ is computed (820) as a function of the received data symbols 815 and the symbol estimates 810. This is such that $z_{0,b} = D^H X^H(b) Y(b)$ where:

$$(K \times JK) X(b) = [X_1(b) \dots X_J(b)], JK \times JL D = \begin{bmatrix} F & 0 & \dots & 0 \\ 0 & F & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & F \end{bmatrix}$$

$(K \times K) X_j(b) = \text{diag}(x_j(1,b), \dots, x_j(K,b))$ Diag means the matrix is composed of all zeros except for the diagonal elements which are given in parenthesis. $x_j(1,b)$ is user j 's symbol estimate on subcarrier k and block b . For the matrix above, F is determined by:

$$(K \times L) F = \begin{bmatrix} f^H(0) \\ \vdots \\ f^H(K-1) \end{bmatrix}, \text{ where } (L \times 1) f(k) = \begin{bmatrix} 1 \\ \text{e}^{j2\pi k/L} \\ \vdots \\ \text{e}^{j2\pi (L-1)k/L} \end{bmatrix}$$

(In the preferred embodiment, $z_{0,b} = D^H X^H(b) Y(b)$, would be computed by FFT's and an element-wise multiplication of two vectors). Next (825) the iteration number t , is set to zero. The initialization also requires that the channel estimate at iteration number 0, θ_0 , be equal to an initial channel estimate. In the preferred embodiment, the initial channel estimate is equal to the channel estimate at block $b-1$, $h(b-1)$ (830). This is the channel estimate that has been previously computed for the block 790 if block $b-1$ is a pilot block and 890 if block $b-1$ is a data block. In block 835, the gradient vector g is computed at iteration number 0, forming g_0 as a function of the initial time vector, the symbol estimates on block b , and the channel estimate at iteration number 0. The equation for 835 is $g_0 = D^H X^H(b) X(b) D \theta_0 - z_{0,b}$. Then the direction vector d is computed at iteration number 0, forming d_0 as the negative of the gradient vector at iteration number 0 (840). The permuted direction vector v is computed at iteration number 1, as a function of the direction vector at iteration number 1 and the symbol estimates on block b forming v_p (845). The full equation for 845 is $v_p = D^H X^H(b) X(b) D d_1$. Utilizing the equation

$$\alpha_1 = -\frac{g_1^H d_1}{d_1^H d_1}$$

the first step size α is computed at iteration number t , as a function of the direction vector at iteration number t , the gradient vector at iteration number t , and the permuted direction vector at iteration number t forming α_1 (850). Block 855 computes the channel estimate at iteration number $t+1$, as a function of the direction vector at iteration number t , the channel estimate at iteration number t , and the first step size at iteration number t , forming θ_{t+1} , using the equation $\theta_{t+1} = \theta_t + \alpha_t d_t$. Next the gradient vector g is computed at iteration number $t+1$ as a function of the initial time vector, the symbol estimates on block b , and the channel estimate at iteration number t where $g_{t+1} = D^H X^H(b) X(b) D \theta_{t+1} - z_{t+1,b}$ (860). The second step size β is computed at iteration

number i as a function of the direction vector at iteration number i , the gradient vector at iteration number $i+1$, and the permuted direction vector at iteration number i forming β_i (865). This is illustrated as

$$\beta_i = \frac{\partial^2 H}{\partial \theta^2} \theta_i$$

Block 870 computes the direction vector at iteration number $i+1$, θ_{i+1} , as a function of the gradient vector at iteration number $i+1$, the second step size at iteration number i , and the direction vector at iteration number i using the equation $\theta_{i+1} = -\alpha_i \beta_i + \theta_i$. At this point the iteration number i is increased by one (875). If the iteration number i equals i_{max} (880), i_{max} being the number where the channel estimator should stop iterating, the channel estimate on block b , $h(b)$, is the channel estimate at iteration number i (890). i_{max} is predetermined for the receiving device, and is dependent on its processor power and memory limitations. If $d_i^H d_i < \epsilon$ (885), where $d_i^H d_i$ is the magnitude squared of the direction vector and in the preferred embodiment, ϵ is some small percentage (e.g., 1%) of the theta information ($\theta^H \theta$), the channel estimate on block b , $h(b)$, is the channel estimate at iteration number i (890). Otherwise the process from block 845 through 885 is repeated until the conditions i equals i_{max} or $d_i^H d_i < \epsilon$ are met, at which point the channel estimate on block b , $h(b)$, is the channel estimate at iteration i (890).

There is a simpler, less complex approach to the above-mentioned gradient estimators if the end goal is to find channel estimates for multiple transmitting devices when the transmitting devices employ a known specially designed pilot sequence or when there is only a single transmitting device (either with known pilots or symbol decisions).

One example of a possible pilot sequence for transmitting device i is:

$$b_{i,j}(k) = b_j(k) e^{j 2 \pi \theta_i k} \quad (9)$$

Where $b_j(k)$ is an arbitrary pilot sequence. Note that each of the J transmitting devices has the same pilot sequence except for the linear phase term. The phase term allows simple processing to recover each transmitting devices' channel response. The linear phase term means that when the received frequency-domain data, $Y(k,b)$, is divided by the pilot sequence, $b_j(k)$, the noiseless result contains the sum of each transmitting device's channel multiplied by the linear phase term. Because each transmitting device is assumed to have L time taps that characterize its frequency response, we can run a single user version of the gradient approach that assumes JL time taps. Again because of the linear phase term, the first L taps will be the one transmitting device, the second L , another transmitting device and so forth.

Note that in some cases quicker convergence of the gradient algorithms is possible by using a spacing greater than L . In other words, the L of Equation (9) is replaced by γL , where $\gamma > 1$. When the gradient algorithms are applied, in the time-domain, one of the transmitting devices time-domain channel taps will be from 0 to $L-1$, the second will be from γL to $(\gamma+1)L-1$, and so forth.

A mathematical description of the gradient method with the linear phase shift of Equation (9) is now described. The extension to arbitrary phase shift and arbitrary pilot sequence is straight forward and involves modifying the definition of $F(Y)$ and $G(Y)$ given below to accommodate the subcarriers where there are pilots symbols and modifying where the time taps for each of the multiple transmitting devices are found after the IFFT processing.

The first step is to define the noisy channel estimate $K \times 1$ vector, $H_n(b)$, as:

$$H_n(b) = \begin{bmatrix} H_{n,1}(b) \\ \vdots \\ H_{n,K}(b) \end{bmatrix} \quad \text{where } H_{n,i}(b) = \frac{Y_{i,b}}{b_j(k)} \quad (10)$$

Next we define $F\{y\}$ to be the N -point IFFT of the $K \times 1$ vector y truncated to JL samples and scaled by $N/\sqrt{K}$ (for other linear phase shifts, the JL samples kept are the ones that correspond to each of the J transmitting devices time-domain channels whose location after the IFFT is specified by the phase shift). Also we define $G\{x\}$ to be the N -point FFT of the $JL \times 1$ vector x truncated to K samples and scaled by $1/\sqrt{K}$.

Fixed Step Size and Steepest Descent Linear Phase Shift Gradient Method

1. $i=0$, $\theta_0 = F\{H_n(b)\}$
2. $d_i = G\{F\{G\{\theta_i\}\}\}$
 $\theta_{i+1} = \theta_i + \alpha_i d_i$
 where $\alpha_i = 1$ (or any appropriate scalar) for fixed step size gradient and for steepest descent it equals:

$$\alpha_i = \frac{d_i^H d_i}{d_i^H d_i} \quad i_1 = G\{d_i\}$$

3. $i=i+1$. If $i > i_{max}$ then stop and $h(b) = \theta_i$, else go to 2

Conjugate Linear Phase Shift Gradient Method

1. $i=0$, $\theta_0 = F\{H_n(b)\}$
2. $g_0 = F\{G\{\theta_0\}\} - \theta_0$, $d_0 = -g_0$

$$3. \alpha_i = -\frac{g_i^H d_i}{d_i^H d_i} \quad i_1 = F\{G\{d_i\}\}$$

4. $\theta_{i+1} = \theta_i + \alpha_i d_i$
5. $g_{i+1} = F\{G\{\theta_{i+1}\}\} - \theta_{i+1}$

$$6. \beta_i = \frac{g_{i+1}^H g_{i+1}}{d_i^H d_i}$$

7. $d_{i+1} = -g_{i+1} + \beta_i d_i$

8. $i=i+1$. If $i > i_{max}$ then stop and $h(b) = \theta_i$, else go to 3

This description can also be implemented in FIGS. 6 through 9 providing the steps within the corresponding blocks of the figures are carried out in accordance with the above description.

The present invention may be embodied in other specific forms without departing from its spirit or essential characteristics. The described embodiments are to be considered in all respects only as illustrative and not restrictive.

We claim:

1. A method of operating a communication system including at least one receiver comprising:
 - providing a plurality of pilot symbols;
 - receiving pilot data;
 - determining an initial time vector as a function of the received pilot data and pilot symbols;
 - determining an initial direction vector as a function of the initial time vector and the pilot symbols; and
 - determining a first channel estimate as a function of the initial direction vector, the initial time vector, and an initial step size.

2. The method of claim 1 wherein the initial time vector is computed using Discrete Fourier Transform (DFT) processing.

3. The method of claim 2 wherein the Discrete Fourier Transform (DFT) comprises a Fast Fourier Transform (FFT).

4. The method of claim 1 further comprising determining a second channel estimate as a function of a second direction vector, the first channel estimate and a second step size.

5. The method of claim 1 further comprising determining an initial gradient vector from the initial direction vector, wherein the first channel estimate is determined as a function of the initial gradient vector.

6. A method of operating a communication system including at least one receiver comprising:

providing a plurality of symbol estimates;

receiving a plurality of data symbols;

determining an initial time vector as a function of the received data symbols and symbol estimates;

determining an initial direction vector as a function of the initial time vector, the symbol estimates, and an initial channel estimate; and

determining a first channel estimate as a function of the initial direction vector, the initial channel estimate and an initial step size.

7. The method of claim 6 wherein the initial time vector is computed using Discrete Fourier Transform (DFT) processing.

8. The method of claim 6 wherein the Discrete Fourier Transform (DFT) comprises a Fast Fourier Transform (FFT).

9. The method of claim 6 further comprising determining a second channel estimate as a function of a second direction vector, the first channel estimate and a second step size.

10. The method of claim 6 wherein the initial channel estimate is equal to a channel estimate from a previous time.

11. The method of determining channel estimation for a pilot block between a transmitting device and a receiving device comprising:

a) initializing a channel estimation device by storing pilot symbols;

b) receiving pilot data;

c) computing an initial time vector as a function of a received pilot data block and pilot symbols;

d) initializing an iteration number t to zero;

e) initializing a channel estimate to equal the initial time vector;

f) computing a direction vector based on the initial time vector, pilot symbols, and channel estimate at iteration t ;

g) computing a channel estimate at iteration $t+1$ based on the direction vector at iteration t , a channel estimate at iteration t , and a step size at iteration t ;

h) incrementing iteration number by 1; and

i) if t is not equal to a preset number, then repeat steps f) through i); and

j) if a magnitude squared of the direction vector is not less than a predetermined value, repeat steps f) through j).

12. The method of determining channel estimation for a data block between a transmitting device and a receiving device comprising:

a) initializing a channel estimation device by determining and storing symbol estimates;

b) receiving data symbols;

c) computing an initial time vector as a function of a received data symbols and symbol estimates;

d) initializing an iteration number t to zero;

e) initializing a channel estimate to equal a channel estimate from a previous time;

f) computing a direction vector based on a initial time vector, pilot symbols, and channel estimate at iteration t ;

g) computing a channel estimate at iteration $t+1$ based on the direction vector at iteration t , a channel estimate at iteration t , and a step size at iteration t ;

h) incrementing iteration number by 1;

i) if t is not equal to a preset number, then repeat steps f) through i); and

j) if a magnitude squared of the direction vector is not less than a predetermined value, repeat steps f) through j).

13. A receiver for a communication system comprising:

means for receiving a plurality of pilot symbols;

means for receiving pilot data;

means for determining an initial time vector as a function of the received pilot data and pilot symbols;

means for determining an initial direction vector as a function of the initial time vector and the pilot symbols; and

means for determining a first channel estimate as a function of the initial direction vector, the initial time vector, and an initial step size.

14. The receiver of claim 13 wherein the initial time vector is computed using Discrete Fourier Transform (DFT) processing.

15. The receiver of claim 13 wherein the Discrete Fourier Transform (DFT) comprises a Fast Fourier Transform (FFT).

16. The receiver of claim 13 further comprising means for determining a second channel estimate as a function of a second direction vector, the first channel estimate and a second step size.

17. A receiver for a communication system comprising:

means for receiving a plurality of symbol estimates;

means for receiving a plurality of data symbols;

means for determining an initial time vector as a function of the received data symbols and symbol estimates;

means for determining an initial direction vector as a function of the initial time vector, the symbol estimates, and an initial channel estimate; and

means for determining a first channel estimate as a function of the initial direction vector, the initial channel estimate and an initial step size.

18. The receiver of claim 17 wherein the initial time vector is computed using Discrete Fourier Transform (DFT) processing.

19. The receiver of claim 17 wherein the Discrete Fourier Transform (DFT) comprises a Fast Fourier Transform (FFT).

20. The receiver of claim 17 further comprising means for determining a second channel estimate as a function of a second direction vector, the first channel estimate and a second step size.

21. The receiver of claim 17 wherein the initial channel estimate is equal to a channel estimate from a previous time.

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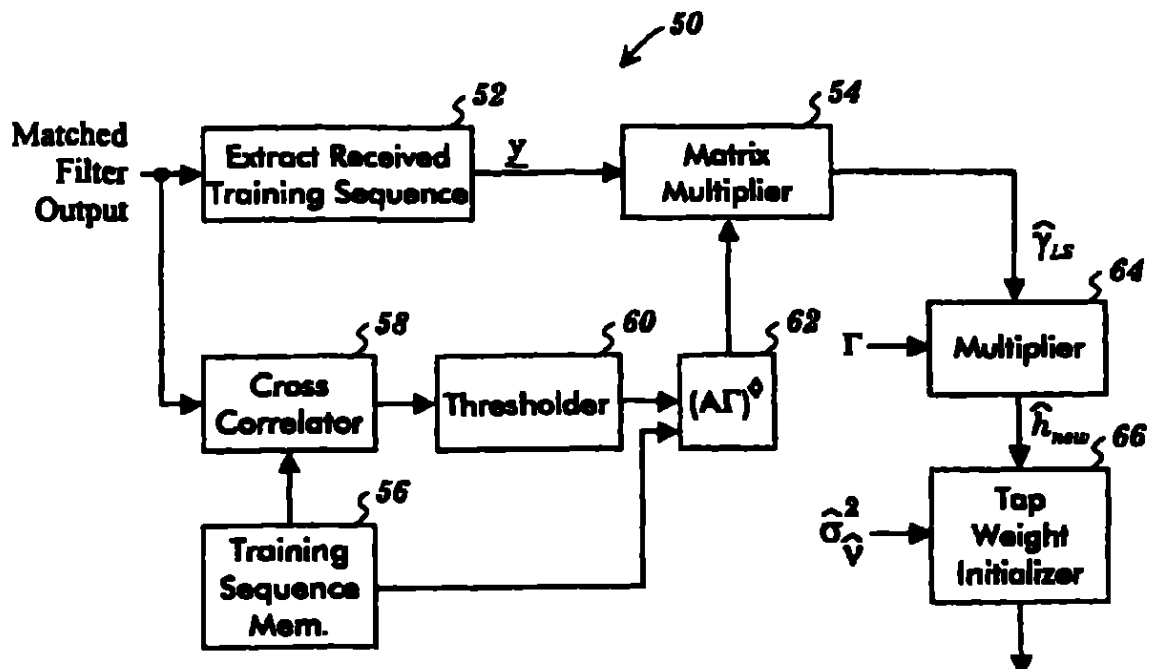
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(54) Title: CHANNEL ESTIMATION METHOD BLENDING CORRELATION AND LEAST-SQUARES BASED APPROACHES



(57) Abstract: A response of a channel may be estimated by correlating a received signal and a training sequence, by forming a matrix Γ based on a desired shape for the peaks of the correlation, by extracting a vector y from the received signal, and by estimating the channel response from a least-squares solution based on the matrix Γ , the vector y , and a matrix formed from the elements of the known training sequence.

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CHANNEL ESTIMATION METHOD BLENDING CORRELATION AND LEAST-SQUARES BASED APPROACHES

Related Applications

The present application claims the benefit of Provisional Application Serial No. 60/336,415 filed on October 24, 2001.

5

Technical Field of the Invention

The present invention relates to a channel estimator and, more particularly, to a channel estimator that estimates the response of a channel through which signals are received.

10

Background of the Invention

For communication systems utilizing a periodically transmitted training sequence, least-squares algorithms or correlation algorithms have been the most widely used alternatives for the estimation of the channel. The estimation of the channel may then be used for various purposes such as initializing the values of the taps of an equalizer at a receiver.

20

Both algorithms use a stored copy of the known transmitted training sequence at the receiver. The properties and length of the training sequence generally depend on the particular standard specifications of the communication system. In addition, a training sequence

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is also used in a communication system for synchronization to indicate the beginning and/or end of a transmitted data frame or packet.

In general, a communication system 10 that
5 utilizes an equalizer adjusted by use of the training signal is shown in Figure 1 and includes an equalizer 12 and a channel estimator 14 of a receiver. The base-band transmitted signal $s(t)$ having a data rate of $1/T$ symbols per second is represented by the following equation:

10

$$s(t) = \sum_k I_k q(t - kT) \quad (1)$$

where $I_k \in A = \{\alpha_1, \dots, \alpha_M\} \subset \mathbb{C}$ is the transmitted training sequence, which is a discrete M-ary sequence taking
15 values on the generally complex M-ary alphabet A, which also constitutes a two dimensional modulation constellation. The function $q(t)$ as shown in a block 16 represents the pulse shaping performed by a transmitter filter of finite support $[-T_q/2, T_q/2]$.

20 The overall complex pulse shape is denoted by $p(t)$ and is given by the following equation:

$$p(t) = q(t) * q^*(-t) \quad (2)$$

where $q^*(-t)$ shown in a block 18 is the receiver matched filter finite impulse response. Although it is not required, it is assumed, for the sake of simplifying the notation, that the span, T_q , of the transmitter and receiver filters is an integer multiple of the symbol period T ; that is, $T_q = N_q T, N_q \in \mathbb{Z}^+$.

The physical channel between the transmitter and the receiver is denoted by $c(t, \tau)$ and is shown by a block 20, and throughout the discussion below the concatenation of $p(t)$ and the physical channel $c(t, \tau)$ will be denoted by $h(t)$ and is defined by the following equation:

$$h(t) = q(t) * c(t, \tau) * q^*(-t) = p(t) * c(t, \tau) \quad (3)$$

where $h(t)$ is generally referred to as the channel response. The physical channel $c(t, \tau)$ between the transmitter and the receiver is generally described by the following impulse response:

$$c(t, \tau) = \sum_{k=-K}^L c_k(\tau) \delta(t - \tau_k) \quad (4)$$

which describes a time-varying channel, where $\{c_k(\tau)\} \subset C^1$,
 $-K \leq k \leq L$, and $t, \tau \in R$. However, it is assumed that the
time-variation of the channel is slow enough that
 $c(t, \tau) = c(t)$ can be assumed to be a fixed (static) inter-
5 symbol interference (ISI) channel throughout the training
period; that is, it is assumed that $c_k(\tau) = c_k$, which in
turn implies that $c(t)$ is defined by the following
equation:

$$10 \quad c(t) = \sum_{k=-K}^L c_k \delta(t - \tau_k) \quad (5)$$

for $0 \leq t \leq NT$, where N is the number of training symbols.

The summation indices K and L , where K is the
number of anti-causal multi-path delays and L is the
number of causal multi-path delays, are dependent on the
15 channel and are more fully explained below in the
discussion of correlation based channel estimation and
subsequently. The multi-path delays τ_k are not assumed
to be at integer multiples of the sampling period T .
Indeed, an accurate and robust way of recovering the
20 pulse shape back into the concatenated channel estimate
when the multi-path delays are not exactly at the
sampling instants is disclosed herein.

Equations (3) and (5) can be combined according to the following equation:

$$h(t) = p(t) * c(t) = \sum_{k=-K}^L c_k p(t - \tau_k). \quad (6)$$

5

Because both $p(t)$ and $c(t)$ are complex functions, the overall channel impulse response $h(t)$ is also complex. Therefore, $p(t)$, $c(t)$, and $h(t)$ can be written as functions of their real and imaginary parts as given by

10 the following equations:

$$p(t) = p_I(t) + jp_Q(t) \quad (7)$$

$$c(t) = c_I(t) + jc_Q(t) \quad (8)$$

$$h(t) = h_I(t) + jh_Q(t) \quad (9)$$

15

Substituting equations (7)-(9) into equation (6) produces the following equation:

$$\begin{aligned} h(t) &= p(t) * c(t) = (p_I(t) + jp_Q(t)) * (c_I(t) + jc_Q(t)) \\ 20 \quad &= \underbrace{(p_I(t) * c_I(t) - p_Q(t) * c_Q(t))}_{h_I(t)} + \underbrace{j(p_I(t) * c_Q(t) + p_Q(t) * c_I(t))}_{h_Q(t)} \quad (10) \end{aligned}$$

The output matched filter 18 is denoted $y(t)$ and is given by the following equation:

$$y(t) = \left(\sum_k I_k \delta(t - kT) \right) * h(t) + \tilde{v}(t) \quad (11)$$

5

where $\tilde{v}(t)$ designates the complex noise process after the matched filter. As in the case of equations (7)-(9), $y(t)$ can be written in terms of its real and imaginary parts as $y(t) = y_r(t) + jy_i(t)$. Sampling the output of the

10 matched filter 18 at the symbol rate using a sampler 22 produces the discrete-time representation of the overall communication system according to the following equation:

$$y[n] \equiv y(t)|_{t=nT} = \sum_k I_k h[n-k] + \tilde{v}[n]. \quad (12)$$

15

The transmitted symbols are composed of frames (or packets) of length N' , where the first N symbols are the training symbols a_k . Within a frame of length N' , the symbols may be denoted by the following equation:

20

$$I_k = \begin{cases} a_k, & \text{for } 0 \leq k \leq N-1 \\ d_k, & \text{for } N \leq k \leq N'-1 \end{cases} \quad (13)$$

where the distinction of the first N symbols is made to indicate that they are the known training symbols, and where it is possible that the training symbols a_k belong to a subset of the M -ary constellation alphabet A ; that is $\{a_k \in \tilde{A} \subset A \equiv \{\alpha_1, \dots, \alpha_M\}\}$. In fact, for the 8-VSB digital television system, the signal alphabet is $A \equiv \{\pm 1, \pm 3, \pm 5, \pm 7\}$, while the training sequence can only take binary values within the subset $\tilde{A} \equiv \{-5, +5\}$.

Figure 2 shows a least-squares channel estimator 30 that has been used for the channel estimator 14 of Figure 1. In order to fully estimate the channel defined by equation (6), the least-squares based channel estimation algorithm assumes that the starting point and the ending point of the channel taps are either known or can be bounded. This assumption plays a critical role in the overall quality and robustness of the least-squares estimation procedure. It can be equivalently assumed that the impulse responses of the discrete-time equivalent composite channel $h[n]$ can be written as a finite dimensional vector according to the following equation:

$$\mathbf{h} = [h[-N_a], h[-N_a+1], \dots, h[-1], h[0], h[1], \dots, h[N_c-1], h[N_c]]^T \quad (14)$$

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where N_a denotes the number of anti-causal taps of the channel, N_c denotes the number of causal taps of the channel, and $N_a + N_c + 1$ is the total memory of the channel.

5 Based on equation (12) and assuming that $N \geq N_a + N_c + 1$ where N again is the number of training symbols, the output of the matched filter 18 corresponding to the known training symbols can be written as follows:

10

$$\begin{aligned}
 y[N_c] &= h[-N_a]a_{N_c+N_c} + \dots + h[0]a_{N_c} + \dots + h[N_c]a_0 + \bar{u}[N_c] \\
 y[N_c + 1] &= h[-N_a]a_{N_c+N_c+1} + \dots + h[0]a_{N_c+1} + \dots + h[N_c]a_1 + \bar{u}[N_c + 1] \\
 &\vdots \\
 y[N - 1 - N_a] &= h[-N_a]a_{N-1} + \dots + h[0]a_{N-1-N_a} + \dots + h[N_c]a_{N-1-N_a-N_c} + \bar{u}[N - 1 - N_a]
 \end{aligned}$$

15

which can be compactly written according to the following vector equation:

20

$$y = Ah + \bar{u} \quad (15)$$

where y comprises values extracted by a sample extractor 32 from the received signal and is defined according to the following equation:

$$y = [y[N_c], y[N_c + 1], \dots, y[N - 1 - N_a]]^T \quad (16)$$

where A is based on the training signal and is defined according to the following equation:

$$A = \begin{bmatrix} a_{N_c+N_a} & a_{N_c+N_a-1} & \dots & a_0 \\ a_{N_c+N_a+1} & a_{N_c+N_a} & \dots & a_1 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1} & a_{N-2} & \dots & a_{N-1-N_a-N_c} \end{bmatrix}_{(N-N_a-N_c) \times (N_a+N_c+1)} \quad (17)$$

and where $\bar{v}$ is defined according to the following equation:

$$\bar{v} = [\bar{v}[N_c], \bar{v}[N_c + 1], \dots, \bar{v}[N - 1 - N_a]]^T.$$

As long as the matrix A is a tall matrix and of full column rank such that $N \geq 2(N_a + N_c) + 1$ and $\text{rank}\{A\} = N_a + N_c + 1$, then the least-squares solution which minimizes the objective function given by the following equation

$$J_{LS}(h) = \|y - Ah\|^2 \quad (18)$$

exists and is unique and is given by the following equation:

5

$$\hat{h}_{LS} = (A^H A)^{-1} A^H y \quad (19)$$

where the superscript H denotes Hermitian transpose, and where $\hat{h}_{LS}$ is the least-squares estimation of the channel between the transmitter and the receiver. Thus, in accordance with equation (19), a matrix multiplier 34 multiplies the matrix y by a matrix A^+ , which is $(A^H A)^{-1} A^H$ and which is stored in a memory 36. Thus, the matrix multiplier produces the channel estimate $\hat{h}_{LS}$ according to equation (19).

One of the problems associated with the least-squares based channel estimation is the uncertainty associated with the actual channel spread; i.e., the spread between the outermost multi-paths. That is, this uncertainty arises because the channel memory and the first and last channel taps are not actually known by the receiver. Therefore, in order to set up equations (16)-(19), some sort of preprocessing is needed, such as

correlating the stored training symbols with the received signal, or some pre-assumed starting and ending points for the channel must be used such that the total channel spread is not too long that would otherwise result in the
5 A matrix being a wide matrix rather than a tall one.

Even if the starting and ending points of the actual channel spread are known by the receiver, least-squares based channel estimators still have problems that may be insurmountable. For example, least-squares
10 channel estimators cannot properly estimate the channel from the training sequence if the channel spread is too long for the training sequence to effectively support the channel.

That is, if $N < 2(N_a + N_c) + 1$, such that the number
15 of training symbols is insufficient to support, or create, an over-determined channel convolution matrix A , then there are two cases to consider.

First, if the full A matrix as given by equation (17) is used, the least-squares estimate of
20 equation (19) no longer exists. That is, the matrix $A^H A$ is singular.

Second, if the receiver does not attempt to construct the full A matrix and instead constructs an alternative thinner matrix $\tilde{A}$ with $N \geq 2(\tilde{N}_a + \tilde{N}_c) + 1$ such

that $\tilde{N}_a < N_a$ and/or $\tilde{N}_c < N_c$, then the inverse of $\tilde{A}^H \tilde{A}$ will exist but the new least-squares solution given by the following equation:

$$\tilde{h}_{LS} = (\tilde{A}^H \tilde{A})^{-1} \tilde{A}^H y$$

will not be a very desirable estimate. That is, the resulting channel estimate no longer resembles the actual channel, especially if the actual channel spread is much longer than the training sequence can support.

An example is illustrated in Figures 3(a)-3(e). A channel with delays $\tau_k = [-105.32, -87.643, -45.443, 0, 12.22, 39.9, 203.659, 321.82, 435.12]$ with a complex raised cosine pulse shape of 41 samples (a channel spread of 580 symbols including the tails of the pulse shape) was simulated. The real part of the channel impulse response seen at the output of the matched filter is shown in Figure 3(a). As shown in Figure 3(a), each peak of the channel impulse response has a raised cosine pulse shape. The training sequence is the same as in the 8 VSB digital television standard and is of length $N = 704$ symbols. With 704 training symbols, a channel with a maximum delay spread of only $N_a + N_c + 1 = 352$ taps, including the tails of the pulse shape, can be supported.

Thus, the actual channel has a much longer delay spread than the standard least-squares based method can handle. The least-squares channel estimate corresponding to cases where the receiver constructs a smaller convolution matrix $\tilde{\mathbf{A}}$ with $-\tilde{N}_s = -125$, $\tilde{N}_e = 60$, $-\tilde{N}_s = -125$, $\tilde{N}_e = 120$, $-\tilde{N}_s = -125$, $\tilde{N}_e = 206$, and $-\tilde{N}_s = -125$, $\tilde{N}_e = 342$ are shown in Figure 3(b), Figure 3(c), Figure 3(d), and Figure 3(e), respectively. The matrix $\tilde{\mathbf{A}}$ is non-singular for all cases except Figure 3(e). As can be seen from Figure 3(b) to Figure 3(d), as the assumed channel spread gets longer, the estimated channel response increasingly deviates from the original channel response. In Figure 3(e), where $\tilde{\mathbf{A}}$ is undetermined, the estimated channel no longer resembles the original channel response. However, it may be misleading to assume a very short channel spread in order to obtain a cleaner channel estimate within the assumed channel span. In that case, it is very likely that very significant early pre-cursor multi-paths or late post-cursor multi-paths may be missing.

As indicated above, it is also known to apply auto-correlation in estimating the channel. The sampled received sequence at the output of the matched filter is denoted by $y[n]$ and is given in equation (12), where the pre-cursor symbols are at positions $n \geq 0$ and post-cursor

symbols are at positions $n < 0$. The known training sequence is denoted by $s[n] = a_n$ for $n = 0, \dots, N-1$. The auto-correlation functions $r_s[m]$ and $r_{sy}[m]$ are defined in accordance with the following equations:

5

$$r_s[m] = \sum_{k=0}^{N-1} s^*[k]s[k+m] \quad (20)$$

$$r_{sy}[m] = \sum_{k=0}^{N-1} s^*[k]y[k+m] \quad (21)$$

where $\{s[k] \in \tilde{A}\}$.

10

It is assumed that, in order to be able to use correlation based channel estimation, the training sequence must belong to a certain class of sequences and thereby possess certain desirable properties. An exemplary class of sequences is maximal length pseudo-noise (PN) sequences. A PN-sequence of length n is denoted herein as PN_n . In general, the periodic auto-correlation of a binary valued ($\{+A, -A\}$) PN_n sequence is given by the following equation:

20

$$r_{PN_n}[m] = \begin{cases} A^2 n, & \text{if } m = 0, \pm n, \pm 2n, \dots \\ -A^2, & \text{otherwise} \end{cases} \quad (22)$$

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Figure 4 shows a correlation channel estimator 40 that can be used for the channel estimator 14 to estimate the channel in order to initialize the equalizer 12. The correlation channel estimator 40 includes a cross-correlator 42 that cross-correlates the received signal with the training sequence stored in a memory 44 and a level thresholder 46 that sets to zero any correlation coefficients that are below a predetermined threshold. Accordingly, the correlation channel estimator 40 produces a channel estimate $\hat{h}$.

More specifically, based on the auto-correlation property of equation (22), the channel can be estimated by cross-correlating $s[n]$ (the training sequence known and stored in the receiver) with the received sequence $y[n]$. The initial pre-cursor impulse response estimate $\tilde{h}_a[n]$ is determined from the cross-correlation of the training sequence $s[n]$ stored in the receiver and the actual received symbols at lags from 0 to $-N+1$ with respect to the start of the received training sequence. That is, $\tilde{h}_a[n]$ is given by the following equation:

$$\tilde{h}_a[n] = \frac{1}{r_s[0]} \sum_{k=0}^{N-1} s^*[k] y[k+n] \quad \text{for lags } n = 0, -1, \dots, -N+1 \quad (23)$$

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Similarly, the initial post-cursor response estimate $\tilde{h}_c[n]$ is determined from the cross-correlation of the training sequence stored in the receiver and the actual received symbols at lags from 1 to $N-1$ with respect to the start of the received training sequence. That is, $\tilde{h}_c[n]$ is given by the following equation:

$$\tilde{h}_c[n] = \frac{1}{r_s[0]} \sum_{k=0}^{N-1} s[k] y^*[k+n] \quad \text{for lags } n=1, 2, \dots, N-1 \quad (24)$$

10

where $r_s[0]$ in equations (23) and (24) is defined according to the following equation:

$$r_s[0] = \sum_{k=0}^{N-1} |a_k|^2$$

15

Equations (23) and (24) imply that the correlations should be extended on either side by the length of the training sequence in order to obtain the longest possible channel estimate within the limits of the transmission standard, with the assumption that $N \geq K$ and $N \geq L$.

20

With respect to the channel estimation procedure given by equations (23) and (24), the overall

initial estimated channel transfer function $\tilde{H}(z)$ is given by the Z-transform of the concatenated channel estimates according to the following equation:

$$\begin{aligned} \tilde{H}(z) &= \sum_{k=-N+1}^0 \tilde{h}_a[k] z^{-k} + \sum_{k=1}^{N-1} \tilde{h}_c[k] z^{-k} \\ &= \sum_{k=-N+1}^{N-1} \tilde{h}[k] z^{-k} \end{aligned} \quad (25)$$

where the concatenated estimated channel impulse $\tilde{h}[n]$ is given by the following equation:

$$\tilde{h}[n] = \begin{cases} \tilde{h}_a[n], & \text{for } -N+1 \leq n \leq 0 \\ \tilde{h}_c[n], & \text{for } 1 \leq n \leq N-1 \end{cases} \quad (26)$$

Equation (25) can be rewritten in accordance with the following equation:

$$\tilde{H}_x(z) = \frac{1}{r_s[0]} \tilde{A}^{-H} \tilde{y} z^{-k} \quad (27)$$

where $\tilde{A}$ is a $(N + N_a + N_c)$ by $(N_a + N_c + 1)$ Toeplitz matrix with a first column $[a_0, a_1, \dots, a_{N-1}, 0, \dots, 0]^T$ and a first row $[a_0, 0; \dots, 0]^T$, and where $\tilde{y} =$

$[y[-N_a], \dots, y[N + N_c - 1]]^T$. In order to get rid of the side lobes of the aperiodic autocorrelation, the autocorrelation of the training sequence is determined according to the following equation:

5

$$R_{aa} = \frac{1}{r_s[0]} \tilde{A}^H \tilde{A} \quad (28)$$

and then R_{aa} can be inverted and applied to $\tilde{H}_u(z)$ according to the following equation:

10

$$\tilde{H}_c(z) = R_{aa}^{-1} \tilde{H}_u(z) \quad (29)$$

However, the channel estimate $\tilde{H}_c(z)$ obtained by equation (29) has the contributions due to unknown symbols prior to and after the training sequence, as well as the additive channel noise; only the side lobes due to the aperiodic correlation is removed.

As discussed below, the correlations of equations (23) and (24) will clearly yield peaks at $\{D_k^a \in Z^+\}$, for $k = -K, \dots, -1, 0$, and at $\{D_k^c \in Z^+\}$, for $k = 1, \dots, L$. These delays D_k^a and D_k^c are the sampling instants closest to the locations of the actual physical

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channel multi-path delays τ_k for $k = -K, \dots, -1, 0, 1, \dots, L$ within a symbol interval.

As shown in Figure 4, there is a level thresholding algorithm following right after the cross-correlations such that the estimated channel taps are set to zero if they are below a certain threshold. This thresholding algorithm operates in accordance with the following equation:

$$10 \quad \text{set } \hat{h}_c[n] = \begin{cases} 0, & \text{if } \tilde{h}_c[n] < \varepsilon \\ \tilde{h}_c[n], & \text{otherwise} \end{cases} \quad (30)$$

for $n = -N+1, \dots, -1, 0, 1, \dots, N-1$, where the subscript c denotes that the corresponding function has been cleansed of side lobes. As a result, the estimated transfer function $\hat{H}_c(z)$ after thresholding can, in general, be written according to the following equation:

$$\begin{aligned} \hat{H}_c(z) &= \tilde{\beta}_K z^{D_K^a} + \dots + \tilde{\beta}_2 z^{D_2^a} + \tilde{\beta}_1 z^{D_1^a} + \tilde{\alpha}_0 + \tilde{\alpha}_1 z^{-D_1^c} + \tilde{\alpha}_2 z^{-D_2^c} + \dots + \tilde{\alpha}_L z^{-D_L^c} \\ &= \tilde{\alpha}_0 (\beta_K z^{D_K^a} + \dots + \beta_1 z^{D_1^a} + 1 + \alpha_1 z^{-D_1^c} + \dots + \alpha_L z^{-D_L^c}) \end{aligned} \quad (31)$$

20

where $\alpha_i = \tilde{\alpha}_i / \tilde{\alpha}_0$ for $1 \leq i \leq L$, and $\beta_k = \tilde{\beta}_k / \tilde{\alpha}_0$ for $1 \leq k \leq K$.

The notation of equation (31) is adopted because, after

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thresholding, there are usually very few dominant taps. Accordingly, the channel after correlation and thresholding is sparse. Moreover, the peaks in the channel estimate no longer have the pulse shapes of the
 5 actual channel response.

In equation (31), the coefficients β_k correspond to the post-cursor (causal) part of the correlation, and the coefficients α_i correspond to the pre-cursor (anti-causal) part of the correlation. The
 10 delays D_k^a are the anti-causal delays, and the delays D_l^c are the causal delays. As a convention, it is assumed that $1 \leq D_1^c < D_2^c < \dots < D_L^c$ and similarly that $1 \leq D_1^a < D_2^a < \dots < D_K^a$. Also, by the construction of the channel estimates in equations (23) and (24), $D_K^a \leq N-1$
 15 and $D_L^c \leq N-1$.

At this point, the causal part $\hat{H}_{cc}$ and the anti-causal part $\hat{H}_{ac}$ of the transfer function of equation (31) can be defined in accordance with the following equations:

20

$$\hat{H}_{ac}(z) = \tilde{\alpha}_0(\beta_K z^{D_K^a} + \dots + \beta_2 z^{D_2^a} + \beta_1 z^{D_1^a} + 1) \quad (32)$$

$$\hat{H}_{ac}(z) = \hat{\alpha}_0(\alpha_1 z^{-D_1^c} + \alpha_2 z^{-D_2^c} + \dots + \alpha_L z^{-D_L^c}) \quad (33)$$

such that $\hat{H}_c(z) = \hat{H}_{ac}(z) + \hat{H}_{cc}(z)$. Again the first subscript denotes whether the part is anti-causal or causal, and
 5 the second subscript denotes that the part has been cleansed of the side lobes. The partition of equations (32) and (33) enable the development of the appropriate feed-forward filter initializations for a Decision Feedback Equalizer (DFE).

10 Unfortunately, if the PN training sequence is finite and the standard linear correlation is used, the auto-correlation values corresponding to the non-zero lags will not be constant and will not be as low as $-A^2$. As a simple illustration, a sequence composed of six PN₅₁₁
 15 appended back to back may be considered so that the sequence is given by the following equation:

$$y = [PN_{511}, PN_{511}, PN_{511}, PN_{511}, PN_{511}, PN_{511}]. \quad (34)$$

20 Then, $r_{xy}[m]$, with $x = [PN_{511}]^T$, will be given as shown in Figure 5. A low correlation value of $-A^2$ will be obtained for lags that are not multiples of $L = 511$ corresponding to the intermediate PN₅₁₁ portions of the of

the long sequence y . However, as illustrated in Figure 5, for the outermost lags, this constant and low correlation value of $-A^2$ will not be obtained. Instead, these correlation tails are noise like due to the finiteness of the training sequence. That is, the stored training sequence is correlated not only with the training sequence in the received signal but also with random data symbols and reserved symbols.

The training sequence that is used for digital television is given by the following equation:

$$\tilde{s} = [ss, PN_{511}, PN_{63}, \pm PN_{63}, PN_{63}]^T$$

where $ss = [+5 \ -5 \ -5 \ +5]$ represents the four segment sync symbols at the beginning of the frame sync segment. There are also reserved frame bits and data bits right before and after the training sequence $\tilde{s}$.

In summary, the correlations of the received signal with the stored training sequence will be noisy because the PN sequences are finite in length so that they will not achieve the desired low correlation values for non-zero lags, and because the span of the cross-correlation includes the known training sequence as well as random data symbols and reserved symbols.

Because of these reasons, the initial estimate of the channel that is acquired from the correlation of the matched filter output with the stored training sequence will not yield a clean and accurate channel estimate. Accordingly, even if the channel tap estimates $\{\alpha_k, \beta_k\}$ are scaled properly, they will not be very close to the actual channel tap values $\{c_{kp}(0)\}$ due to the reasons given above as well as due to the fact that there may also be tap values in the tails of the adjacent multi-paths. Hence, both the amplitudes and phases of the estimated tap values may be incorrect.

Moreover, after the thresholding takes place, the most likely result is that the thresholded correlation will include only the peaks of the channel taps, which implies that the tails of the raised cosine pulse shape that are under the noisy correlation output may be eliminated entirely. This elimination of the tails may lead to critical performance loss in the initialization of the equalizer 12 and a slower convergence onto the correct tap values for the equalizer 12 after initialization.

The present invention mitigates the problems of the least-squares channel estimators and the correlation channel estimators providing a channel estimator that is

a blend of a least-squares channel estimator and a correlation channel estimator.

Summary of the Invention

5 In accordance with one aspect of the present invention, a method for determining a channel response comprises the following: determining a matrix Γ based on a correlation of a received signal and a known training sequence; extracting a vector y from a received signal;
10 and, estimating the channel response from a least-squares solution based on the matrix Γ , the vector y , and a matrix A formed from the elements of the known training sequence.

 In accordance with another aspect of the
15 present invention, a method for determining a channel response comprises the following: determining a matrix Γ based on a correlation of a received signal and a known training sequence; extracting a vector y from a received signal; determining an unknown coefficient vector γ_{LS}
20 according to the following equation:

$$\gamma_{LS} = (\Gamma^H A^H A \Gamma)^{-1} \Gamma^H A^H y$$

wherein the unknown coefficient vector γ_{LS} comprises a least-squares solution of the following equation:

$$y = A\Gamma\gamma + v$$

5

and wherein A comprises a matrix based on the training sequence; and, determining a channel response according to the following equation:

10

$$\hat{h}_{new} = \Gamma\gamma_{LS}.$$

In accordance with still another aspect of the present invention, a method for determining a channel response comprises the following: correlating a received signal with a training sequence; constructing a matrix Γ by approximating a desired pulse shape for each peak of the correlation; constructing a matrix A based on the training sequence; solving a least-squares solution of the following equation in order to derive an unknown coefficient vector γ :

20

$$y = A\Gamma\gamma$$

wherein y is a vector representing a received signal;
and, determining the channel response from the unknown
coefficient vector γ and the matrix Γ .

5 Brief Description of the Drawings

These and other features and advantages will
become more apparent from a detailed consideration of the
invention when taken in conjunction with the drawings in
which:

10 Figure 1 illustrates a generalized system whose
equalizer is initialized by a channel estimator;

Figure 2 illustrates a least-squares channel
estimator that has been used for the channel estimator of
Figure 1;

15 Figures 3(a)-3(e) illustrate an exemplary
simulation useful in explaining problems associated with
the least-squares channel estimator of Figure 2;

Figure 4 illustrates a correlation channel
estimator that has been used for the channel estimator of
20 Figure 1;

Figure 5 illustrates an exemplary simulation
useful in explaining problems associated with the
correlation channel estimator of Figure 4; and,

Figure 6 illustrates a channel estimator according to an embodiment of the present invention that is a blend of a least-squares channel estimator and a correlation channel estimator.

5

Detailed Description

Figure 6 illustrates a blended channel estimator 50 having a sample extractor 52, a matrix multiplier 54, a training sequence memory 56 that stores
10 a training sequence, a cross-correlator 58, a level thresholder 60, a combiner unit 62, a multiplier 64, and a tap weight initializer 66. The sample extractor 52 extracts values y from the output of a receiver's matched filter, and the matrix multiplier 54 multiplies the
15 extracted values y by a matrix $(A\Gamma)^0$, where the matrix $(A\Gamma)^0$ is calculated by the combining unit 62 in the manner described below. The cross-correlator 58 cross-correlates the received signal with the stored training sequence stored in the memory 56 and the resulting cross-
20 correlation is thresholded by the level thresholder 60 so that the combining unit 62 can construct the matrix $(A\Gamma)^0$ from the output of the level thresholder 60 and the training sequence stored in the memory 56.

Thus, the blended estimation procedure illustrated by Figure 6 starts from the exact same steps of the correlation based estimation as outlined by equations (23) and (24), and then the thresholding step is performed on the channel estimate as indicated by equation (27) in order to clean up the noise in the channel estimate. These steps produce the cleaned channel estimate $\hat{H}_c(z)$ shown in Equation (31). In equation (31), the value 1 denotes the main signal and the other values in equation (2) denote multi-paths. Equivalently, the cleaned channel impulse response estimate can be represented as shown in the following equation:

$$\hat{h}_c = \bar{\alpha}_0 [\beta_K, \underbrace{0, \dots, 0}_{D_K^a - D_{K-1}^a - 1 \text{ zeros}}, \beta_{K-1}, \dots, \beta_1, \underbrace{0, \dots, 0}_{D_1^a - 1 \text{ zeros}}, \underbrace{0, \dots, 0}_{D_1^c - 1 \text{ zeros}}, \alpha_1, \dots, \alpha_{L-1}, \underbrace{0, \dots, 0}_{D_L^c - D_{L-1}^c - 1 \text{ zeros}}, \alpha_L] \quad (35)$$

where the zeros represent the spaces between the peaks. The structure and notation of equations (31) and (35) are important to the derivation of the algorithm below.

As indicated above, the pulse shape $p(t)$ is convolved by the physical channel $c(t)$ according to equation (6). As also indicated above, this pulse shape

is lost due to the correlation and thresholding process. Therefore, it is desirable to recover this pulse shape for each of the multi-paths. Accordingly, the shifted and scaled copies of the pulse shape $p(t)$ (shifted by τ_k and scaled by c_k) may be approximated by a linear combination of three pulse shape functions shifted by half a symbol interval ($T/2$). This linear combination is given by the following equation:

$$c_k p(nT - \tau_k) \approx \begin{cases} \sum_{l=-1}^1 \gamma_l^{(k)} p\left(\left(n + D_k^e - \frac{l}{2}\right)T\right) & -K \leq k \leq -1 \\ \sum_{l=-1}^1 \gamma_l^{(k)} p\left(\left(n - D_k^e - \frac{l}{2}\right)T\right) & 1 \leq k \leq L \end{cases} \quad (36)$$

where $\{\gamma_l^{(k)}, K \leq k \leq L\}_{l=-1}^1 \subset \mathbb{C}^1$. The last part of equation (36) contains an approximation of the pulse shape of the transmitter/receiver filter combination. By making this approximation, the tails of the complex pulse shape $p(t)$ can be efficiently recovered even though these tails are buried under the noisy output of the correlation processing and are lost when uniform thresholding is applied.

To arrive at this approximation, three vectors $\mathbf{p}_k$ are introduced for $k=-1, 0, +1$, each containing T

spaced samples of the complex pulse shape $p(t)$ shifted by $kT/2$ (for $k = -1, 0, +1$) according to the following three equations:

$$5 \quad p_{-1} = \left[p\left(-N_q T + \frac{T}{2}\right), \dots, p\left(-T + \frac{T}{2}\right), p\left(\frac{T}{2}\right), p\left(T + \frac{T}{2}\right), \dots, p\left(N_q T + \frac{T}{2}\right) \right]^T \quad (37)$$

$$p_0 = [p(-N_q T), \dots, p(-T), p(0), p(T), \dots, p(N_q T)]^T \quad (38)$$

$$p_1 = \left[p\left(-N_q T - \frac{T}{2}\right), \dots, p\left(-T - \frac{T}{2}\right), p\left(-\frac{T}{2}\right), p\left(T - \frac{T}{2}\right), \dots, p\left(N_q T - \frac{T}{2}\right) \right]^T \quad (39)$$

By concatenating these vectors, a $(2N_q+1) \times 3$ matrix P is
 10 defined according to the following equation:

$$P = [p_{-1}, p_0, p_1] \quad (40)$$

This vector represents shifted and sampled values of a
 15 raised cosine pulse.

A matrix Γ whose columns are composed of the shifted vectors p_k , where the shifts represent the relative delays of the multi-paths. This matrix Γ is given by the following equation:

20

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$$\Gamma = \begin{bmatrix} P & \theta_{(D_K^a - D_{K-1}^a)x3} & & & \\ \theta_{(D_K^a + D_L^c)x3} & P & \ddots & \theta_{D_K^a x3} & \\ & \theta_{(D_L^c + D_{K-1}^a)x3} & & P & \theta_{(D_K^a + D_{L-1}^c)x3} \\ & & \theta_{D_L^c x3} & \ddots & P \\ & & & \theta_{(D_L^c - D_{L-1}^c)x3} & \theta_{(D_K^a + D_L^c)x3} \end{bmatrix}$$

(41)

where Γ is of dimension $(D_K^a + D_L^c + 2N_q + 1)$ by $3(K+L+1)$

5 dependent on the channel spread, and $0_{m \times n}$ denotes an m by

n zero matrix such as a $D_K^a - D_{K-1}^a$ by 3 matrix of zeros.

The received signal vector y and the training sequence vector A are defined by equations (16) and (17),

respectively. Because of the assumption that $q(t)$ spans

10 N_q symbol durations, $q[n]$ has $N_q + 1$ sample points and

$p[n]$ in turn has $2N_q + 1$ samples. Therefore, $N_a = D_K^a + N_q$

and $N_c = D_L^c + N_q$.

An unknown vector γ is defined in accordance

with the following equation:

15

$$\gamma = [\gamma^{(-K)}, \dots, \gamma^{(0)}, \dots, \gamma^{(L)}]^T \quad (42)$$

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where $\gamma^{(k)} = [\gamma_{-1}^{(k)}, \gamma_0^{(k)}, \gamma_1^{(k)}]$ for $-K \leq k \leq L$. The unknown vect r
 γ has coefficients $\{\gamma_{-1}^{(k)}\}$ where $n = -1, 0, 1$, and where $k =$
 $-K, \dots, 0, \dots, L$ and has a length of $3(K+L+1)$.

Similarly to equation (15), the vector y representing the
 5 output of the receiver's matched filter may be given by
 the following equation:

$$y = A\Gamma\gamma + v \quad (43)$$

10 where v is the observed noise vector. Using a least-
 squares solution, the unknown coefficient vector γ can be
 estimated in accordance with the following equation:

$$\hat{\gamma}_{LS} = (\Gamma^H A^H A \Gamma)^{-1} \Gamma^H A^H y = (A\Gamma)^{\dagger} y \quad (44)$$

15

Once the least-squares estimated vector γ_{LS} is obtained,
 the new channel estimate $\hat{h}_{new}$, where the pulse tails are
 recovered, can simply be obtained in accordance with the
 following equation:

20

$$\hat{h}_{new} = \Gamma \hat{\gamma}_{LS} \quad (45)$$

The multiplier 64 multiplies $\hat{\gamma}_{LS}$ from the matrix multiplier 54 by Γ according to equation (45) in order to produce $\hat{h}_{new}$. Equations (44) and (45) may be solved using either a singular value decomposition (SVD) method, by a
 5 conjugate gradient (CG) method, or by any other suitable method.

Once the new estimated channel vector is determined per equation (45), the reconstructed vector at the output of the receiver matched filter can be defined,
 10 based on $\hat{h}_{new}$ according to the following equation:

$$\tilde{y} = A\hat{h}_{new} \quad (46)$$

$$\tilde{y} = A\Gamma\hat{\gamma}_{LS} \quad (47)$$

$$\tilde{y} = A\Gamma(\Gamma^H A^H A \Gamma)^{-1} \Gamma^H A^H y \quad (48)$$

15

The variance of the noise samples at the output of the receiver's matched filter, σ_v^2 , can be estimated based on the following equation:

$$20 \quad \sigma_v^2 = \frac{1}{N - D_K^a - N_L^c} \|\tilde{y} - y\|^2 \quad (49)$$

which arises naturally. The length of the vector $\mathbf{Y}$ is

$$N - D_K^a - D_L^c.$$

The tap weight initializer 66 may then initialize the tap weights of the taps of the equalizer
5 12 using the channel estimate $\hat{h}_{new}$ and the noise variance σ_v^2 in a manner that is known in the art.

When the present invention is used to initialize the taps weights of the taps of an equalizer, the noise variance given by equation (49) and the new
10 channel response given by equation (45) can be combined in any known way so as to produce the initial tap weights for the taps of the equalizer. Alternatively, instead of combining the noise variance given by equation (49) and the new channel response given by equation (45), a small
15 non-zero value noise variance may be assumed and this assumed value may be combined with the new channel response given by equation (45) so as to produce the initial tap weights for the taps of the equalizer.

Modifications of the present invention will
20 occur to those practicing in the art of the present invention. For example, the present invention as described above relies on the use of a training signal. However, other signals, such as data signals, can be used to implement the present invention. Also, although the

invention is specifically shown for use in initializing the taps of an equalizer, the channel estimate produced in accordance with the present invention can be used for other purposes.

5 Accordingly, the description of the present invention is to be construed as illustrative only and is for the purpose of teaching those skilled in the art the best mode of carrying out the invention. The details may be varied substantially without departing from the spirit
10 of the invention, and the exclusive use of all modifications which are within the scope of the appended claims is reserved.

We claim:

- 1 1. A method for determining a channel
2 response comprising:
3 determining a matrix Γ based on a correlation
4 of a received signal and a known training sequence;
5 extracting a vector y from a received signal;
6 and,
7 estimating the channel response from a least-
8 squares solution based on the matrix Γ , the vector y , and
9 a matrix A formed from the elements of the known training
10 sequence.
- 1 2. The method of claim 1 wherein the channel
2 response has a pulse shape, and wherein the matrix Γ is
3 constructed so as to approximate the pulse shape of the
4 channel response.

1 3. The method of claim 2 wherein the matrix Γ
2 comprises three sets of samples derived from the pulse
3 shape, wherein the samples have a period T , wherein the
4 first set comprises the pulse shape samples shifted by
5 $+T/2$, wherein the second set comprises the pulse shape
6 samples not shifted, and wherein the third set comprises
7 the pulse shape samples shifted by $-T/2$.

1 4. The method of claim 3 wherein the pulse
2 shape from which the samples are derived comprises a
3 complex raised cosine pulse shape.

1 5. The method of claim 1 wherein the matrix Γ
2 is constructed with a number of zeros dependent on
3 distances between peaks of the correlation.

1 6. The method of claim 5 wherein the channel
2 response has a pulse shape, and wherein the matrix Γ is
3 constructed so as to approximate the pulse shape of the
4 channel response.

1 7. The method of claim 6 wherein the matrix Γ
2 comprises three sets of samples derived from the pulse
3 shape, wherein the samples have a period T , wherein the
4 first set comprises the pulse shape samples shifted by
5 $+T/2$, wherein the second set comprises the pulse shape
6 samples not shifted, and wherein the third set comprises
7 the pulse shape samples shifted by $-T/2$.

1 8. The method of claim 7 wherein the pulse
2 shape from which the samples are derived comprises a
3 complex raised cosine pulse shape.

1 9. The method of claim 1 wherein the least-
2 squares solution is solved using a singular value
3 decomposition method.

1 10. The method of claim 1 wherein the least-
2 squares solution is solved using a conjugate gradient
3 method.

1 11. The method of claim 1 further comprising
2 determining initial tap weights for an equalizer based on
3 the channel response.

1 12. The method of claim 11 further comprising
2 determining the initial tap weights for the equalizer
3 based on the channel response and a noise variance σ_y^2
4 given by the following equation:
5

$$6 \quad \sigma_y^2 = \frac{1}{N - D_K^a - N_L^c} \|\tilde{y} - y\|^2$$

7
8 wherein D denotes distance between peaks of the
9 correlation, wherein the superscript a denotes a distance
10 of a corresponding anti-causal peak, wherein the
11 superscript c denotes a distance of a corresponding
12 causal peak, wherein K denotes the number of anti-causal
13 peaks, wherein L denotes the number of causal peaks,
14 wherein N denotes a number of training symbols in the
15 training sequence, and wherein $\tilde{y}$ is based on the vector
16 y.

1 13. A method for determining a channel
2 response comprising:
3 determining a matrix Γ based on a correlation
4 of a received signal and a known training sequence;
5 extracting a vector y from a received signal;
6 determining an unknown coefficient vector γ_{LS}
7 according to the following equation:

8

$$9 \quad \gamma_{LS} = (\Gamma^H A^H A \Gamma)^{-1} \Gamma^H A^H y$$

10

11 wherein the unknown coefficient vector γ_{LS} comprises a
12 least-squares solution of the following equation:

13

$$14 \quad y = A \Gamma \gamma + v$$

15

16 and wherein A comprises a matrix based on the training
17 sequence; and,

18 determining a channel response according to the
19 following equation:

20

$$21 \quad \hat{h}_{new} = \Gamma \gamma_{LS}.$$

1 14. The method of claim 13 wherein the channel
2 response has a pulse shape, and wherein the matrix Γ is
3 constructed so as to approximate the pulse shape of the
4 channel response.

1 15. The method of claim 14 wherein the matrix
2 Γ comprises three sets of samples p_{-1} , p_0 , and p_{+1} derived
3 from the pulse shape, wherein the samples have a period
4 T , wherein the first set p_{-1} comprises the pulse shape
5 samples shifted by $+T/2$, wherein the second set p_0
6 comprises the pulse shape samples not shifted, and
7 wherein the third set p_{+1} comprises the pulse shape
8 samples shifted by $-T/2$.

1 16. The method of claim 15 wherein the pulse
2 shape comprises a complex raised cosine pulse shape.

1 17. The method of claim 15 wherein the matrix
2 Γ comprises an approximated pulse shape P formed by
3 concatenating p_{-1} , p_0 , and p_{+1} .

1 18. The method of claim 17 wherein the matrix
2 Γ is given by the following equation:

3

$$4 \quad \Gamma = \begin{bmatrix} P & \theta_{(D_K^a - D_{K-1}^a)x3} & \dots & \theta_{D_K^a x3} \\ \theta_{(D_K^a + D_L^c)x3} & P & & \theta_{(D_K^a + D_{L-1}^c)x3} \\ & \theta_{(D_L^c + D_{K-1}^a)x3} & & P \\ & & \theta_{D_L^c x3} & \dots & \theta_{(D_K^c + D_{L-1}^c)x3} \\ & & & \theta_{(D_L^c - D_{L-1}^c)x3} & P \end{bmatrix}$$

5

6 wherein the zero matrices are dependent on distances D
7 between peaks of the correlation, wherein the superscript
8 a denotes a distance of a corresponding anti-causal peak,
9 wherein the superscript c denotes a distance of a
10 corresponding causal peak, wherein K denotes the number
11 of anti-causal peaks, and wherein L denotes the number of
12 causal peaks.

1 19. The method of claim 18 wherein the pulse
2 shape comprises a complex raised cosine pulse shape.

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1 20. The method of claim 13 wherein the matrix
2 A is given by the following equation:

3

$$4 \quad A = \begin{bmatrix} a_{N_c+N_a} & a_{N_c+N_a-1} & \cdots & a_0 \\ a_{N_c+N_a+1} & a_{N_c+N_a} & \cdots & a_1 \\ \vdots & \vdots & \ddots & \vdots \\ a_{N-1} & a_{N-2} & \cdots & a_{N-1-N_a-N_c} \end{bmatrix}_{(N-N_a-N_c) \times (N_a+N_c+1)}$$

5

6 wherein a_i comprise training symbols, wherein N comprises
7 the number of training symbols, wherein N_a comprises the
8 number of anti-causal peaks in the correlation, and
9 wherein N_c comprises the number of causal peaks in the
10 correlation.

1 21. The method of claim 20 wherein the channel
2 response has a pulse shape, and wherein the matrix Γ is
3 constructed so as to approximate the pulse shape of the
4 channel response.

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1 22. The method of claim 21 wherein the matrix
 2 Γ comprises three sets of samples p_{-1} , p_0 , and p_{+1} derived
 3 from the pulse shape, wherein the samples have a period
 4 T , wherein the first set p_{-1} comprises the pulse shape
 5 samples shifted by $+T/2$, wherein the second set p_0
 6 comprises the pulse shape samples not shifted, and
 7 wherein the third set p_{+1} comprises the pulse shape
 8 samples shifted by $-T/2$.

1 23. The method of claim 22 wherein the pulse
 2 shape comprises a complex raised cosine pulse shape.

1 24. The method of claim 22 wherein the matrix
 2 Γ comprises an approximated pulse shape P formed by
 3 concatenating p_{-1} , p_0 , and p_{+1} .

1 25. The method of claim 24 wherein the matrix
 2 Γ is given by the following equation:

3

$$4 \quad \Gamma = \begin{bmatrix} P & \theta_{(D_K^a - D_{K-1}^a)x3} & \dots & \theta_{D_K^a x3} \\ \theta_{(D_K^a + D_L^c)x3} & P & & \\ & \theta_{(D_L^c + D_{K-1}^a)x3} & \theta_{D_K^c x3} & \\ & & P & \theta_{(D_K^a + D_{L-1}^c)x3} \\ & & \theta_{D_L^c x3} & \dots & \theta_{(D_K^a + D_L^c)x3} \\ & & & \theta_{(D_L^c - D_{L-1}^c)x3} & P \end{bmatrix}$$

5
6 wherein the zero matrices are dependent on distances D
7 between peaks of the correlation, wherein the superscript
8 a denotes a distance of a corresponding anti-causal peak,
9 wherein the superscript c denotes a distance of a
10 corresponding causal peak, wherein K denotes the number
11 of anti-causal peaks, and wherein L denotes the number of
12 causal peaks.

1 26. The method of claim 25 wherein the pulse
2 shape comprises a complex raised cosine pulse shape.

1 27. The method of claim 13 further comprising
2 solving the equation for the unknown coefficient vector
3 λ_L using a singular value decomposition method.

1 28. The method of claim 13 further comprising
2 solving the equation for the unknown coefficient vector
3 λ_L using a conjugate gradient method.

1 29. The method of claim 13 further comprising
2 determining initial tap weights for an equalizer based on
3 the channel response..

1 30. The method of claim 13 further comprising
2 determining the initial tap weights for the equalizer
3 based on the channel response $\hat{h}_{new}$ and a noise variance
4 σ_v^2 given by the following equation:

5

$$6 \quad \sigma_v^2 = \frac{1}{N - D_K^a - N_L^c} \|\tilde{y} - y\|^2$$

7

8 wherein D denotes distance between peaks of the
9 correlation, wherein the superscript a denotes a distance
10 of a corresponding anti-causal peak, wherein the
11 superscript c denotes a distance of a corresponding
12 causal peak, wherein K denotes the number of anti-causal
13 peaks, wherein L denotes the number of causal peaks,
14 wherein N denotes a number of training symbols in the
15 training sequence, and wherein $\hat{y}$ is based on the vector
16 y .

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1 31. A method for determining a channel
2 response comprising:
3 correlating a received signal with a training
4 sequence;
5 constructing a matrix Γ by approximating a
6 desired pulse shape for each peak of the correlation;
7 constructing a matrix A based on the training
8 sequence;
9 solving a least-squares solution of the
10 following equation in order to derive an unknown
11 coefficient vector γ :
12
13 $y = A\Gamma\gamma$
14
15 wherein y is a vector representing a received signal;
16 and,
17 determining the channel response from the
18 unknown coefficient vector γ and the matrix Γ .

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1 32. The method of claim 31 wherein the matrix
2 Γ is based on three sets of samples derived from the
3 desired pulse shape, wherein the samples have a period T ,
4 wherein the first set comprises the pulse shape samples
5 shifted by $+T/2$, wherein the second set comprises the
6 pulse shape samples not shifted, and wherein the third
7 set comprises the pulse shape samples shifted by $-T/2$.

1 33. The method of claim 32 wherein the desired
2 pulse shape comprises a complex raised cosine pulse
3 shape.

1 34. The method of claim 31 wherein the matrix
2 Γ is constructed with a number of zeros dependent on
3 distances between peaks of the correlation.

1 35. The method of claim 31 wherein the
2 determining of the channel response from the unknown
3 coefficient vector γ and the matrix Γ comprises
4 multiplying the matrix Γ by the unknown coefficient
5 vector γ .

1 36. The method of claim 31 wherein the desired
2 pulse shape comprises a complex raised cosine pulse
3 shape.

1 37. The method of claim 31 wherein the solving
2 of the least-squares solution comprises solving the
3 least-squares solution using a singular value
4 decomposition method.

1 38. The method of claim 31 wherein the solving
2 of the least-squares solution comprises solving the
3 least-squares solution using a conjugate gradient method.

1 39. The method of claim 31 further comprising
2 adjusting the tap weights of the taps of the equalizer
3 based on the channel response.

1 40. The method of claim 31 further comprising
2 determining the initial tap weights for the equalizer
3 based on the channel response and a noise variance σ_v^2
4 given by the following equation:

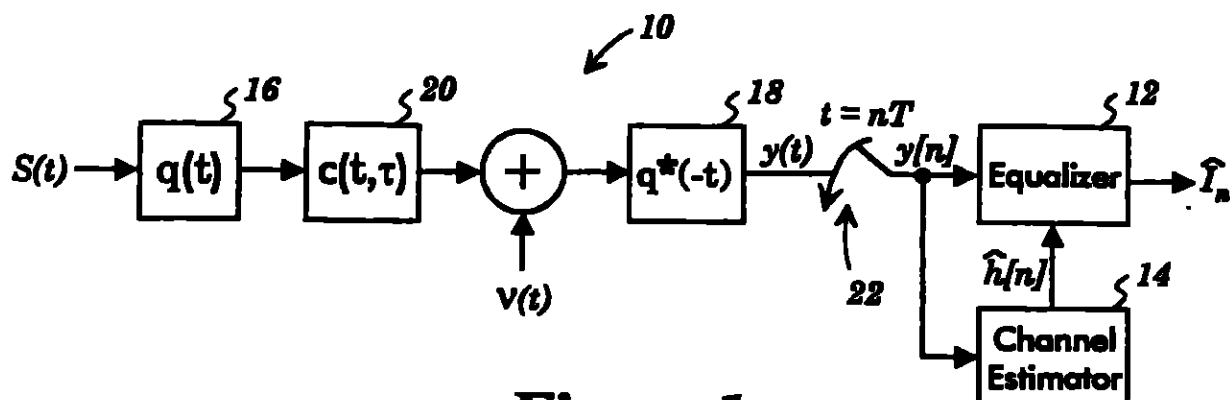
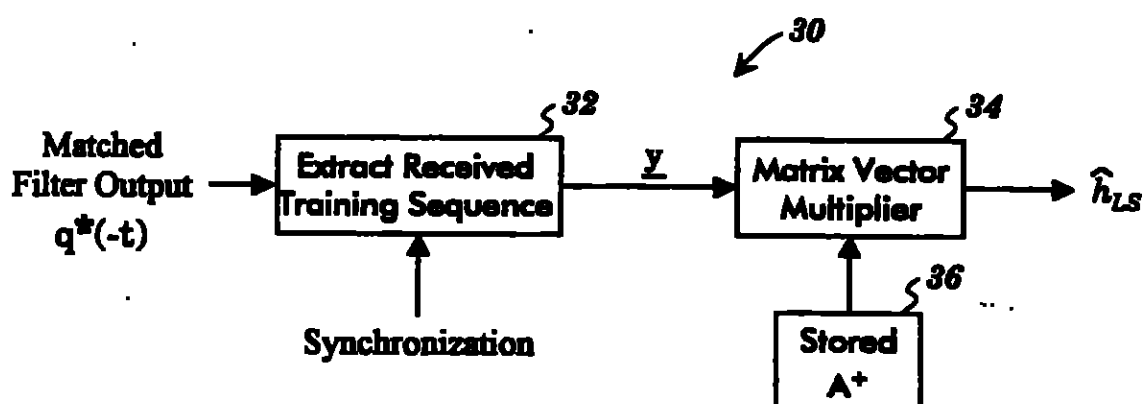
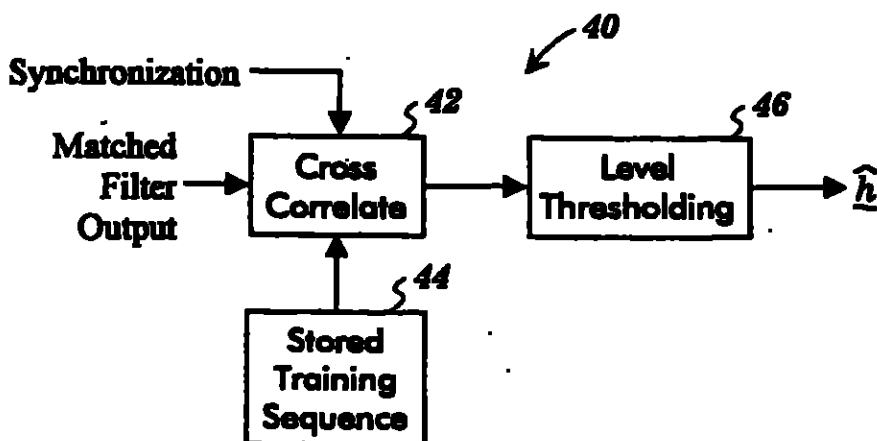
5

$$6 \quad \sigma_v^2 = \frac{1}{N - D_K^a - N_L^c} \|\hat{y} - y\|^2$$

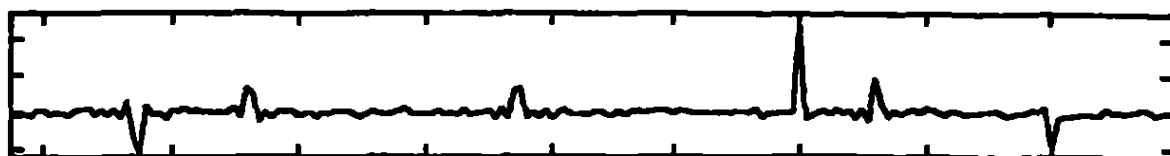
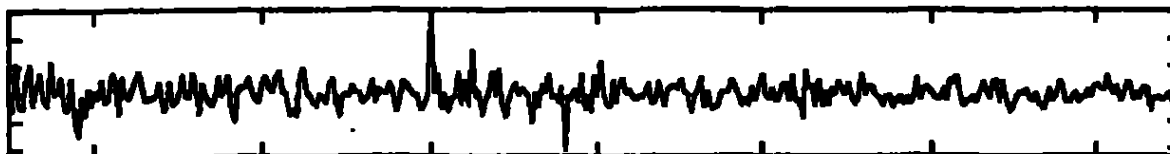
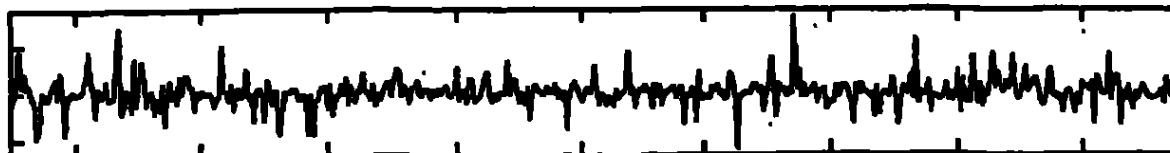
7

8 wherein D denotes distance between peaks of the
9 correlation, wherein the superscript a denotes a distance
10 of a corresponding anti-causal peak, wherein the
11 superscript c denotes a distance of a corresponding
12 causal peak, wherein K denotes the number of anti-causal
13 peaks, wherein L denotes the number of causal peaks,
14 wherein N denotes a number of training symbols in the
15 training sequence, and wherein $\hat{y}$ is based on the vector
16 y.

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**Figure 1****Figure 2****Figure 4**

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*Figure 3(a)**Figure 3(b)**Figure 3(c)**Figure 3(d)**Figure 3(e)*

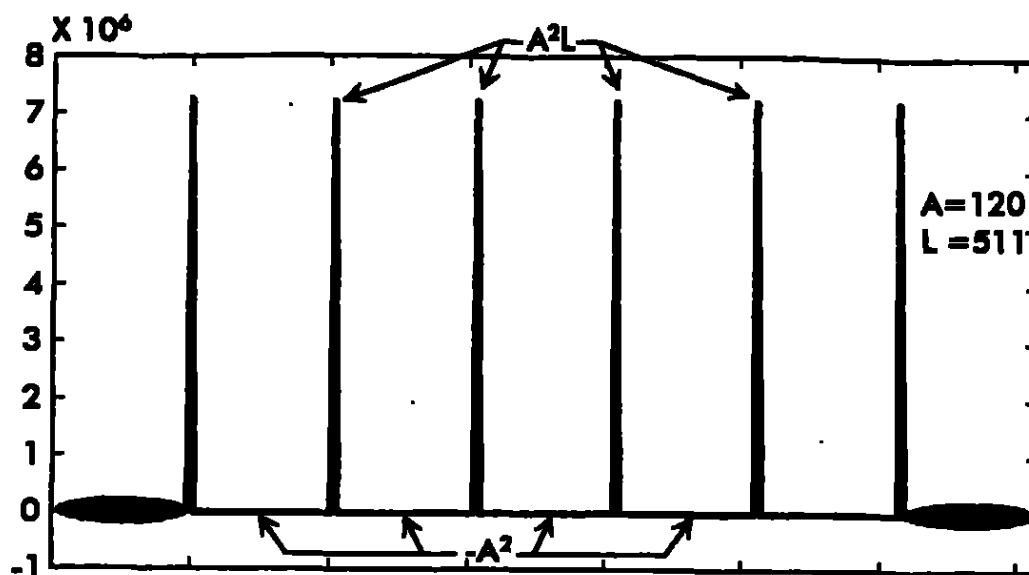


Figure 5

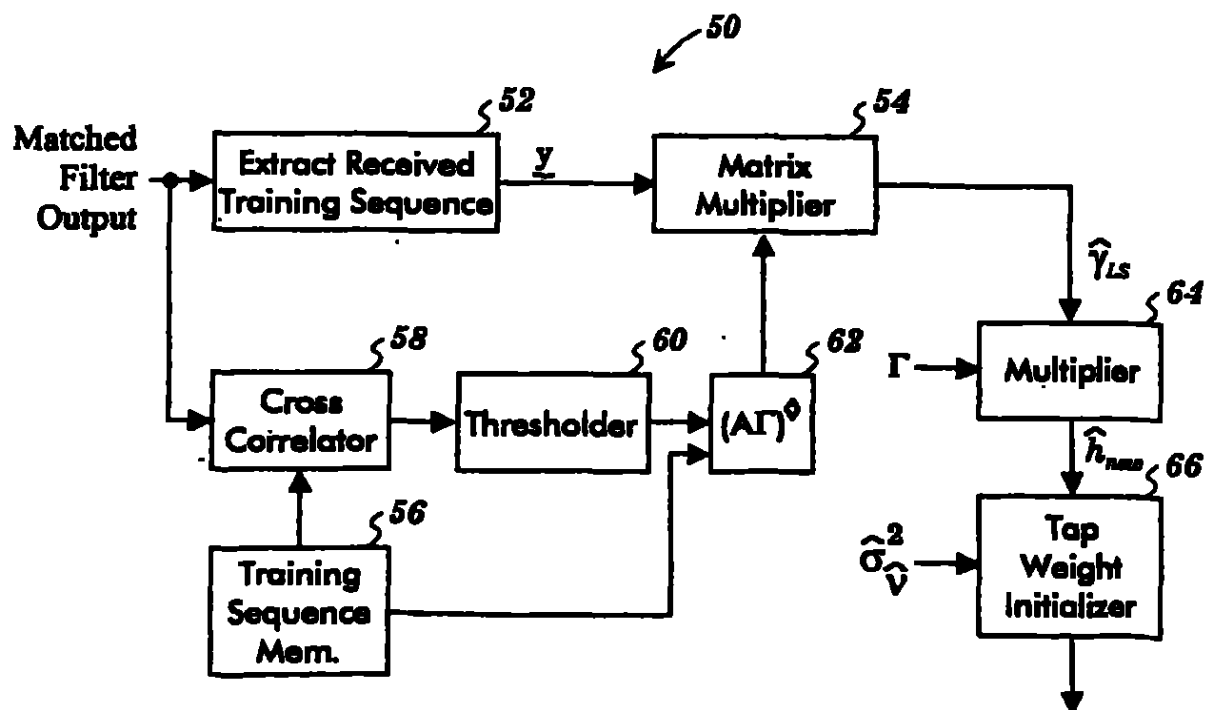


Figure 6

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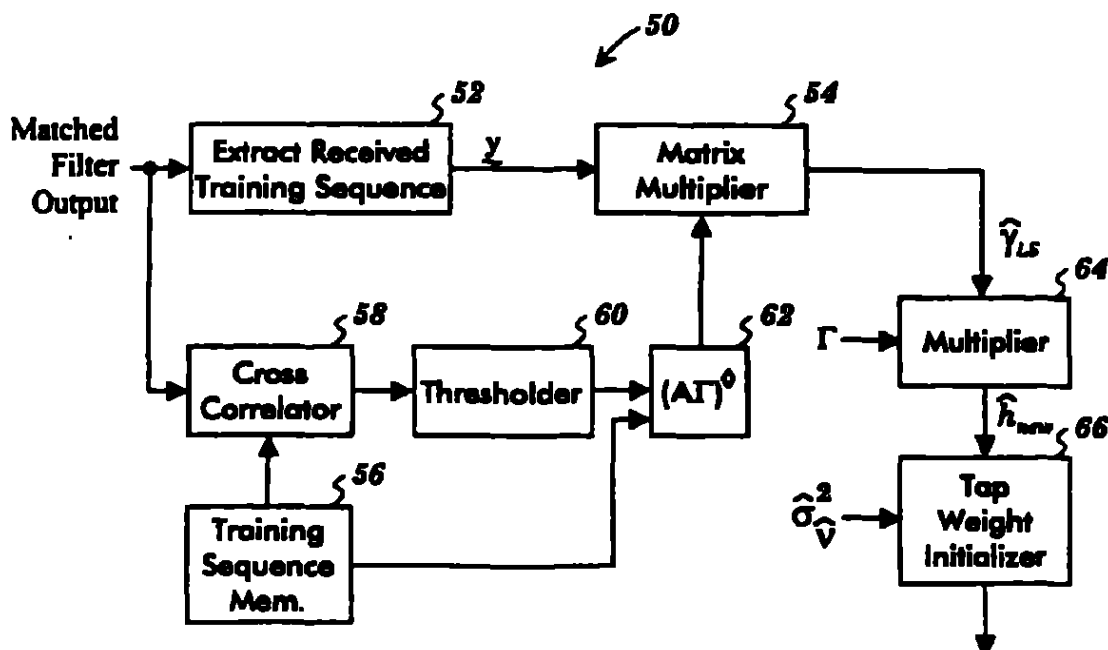
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(54) Title: CHANNEL ESTIMATION USING A COMBINATION OF CORRELATION AND LEAST SQUARE ERROR



(57) Abstract: A response of a channel may be estimated by correlating a received signal and a training sequence, by forming a matrix Γ based on a desired shape for the peaks of the correlation, by extracting a vector y from the received signal, and by estimating the channel response from a least-squares solution based on the matrix Γ , the vector y , and a matrix formed from the elements of the known training sequence.

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Electronic data base consulted during the International search (name of data base and, where practical, search terms used)

EPO-Internal, PAJ, WPI Data, COMPENDEX, INSPEC

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category *	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X A	WO 98 38772 A (WHITE PETER JOHN ;NOKIA TELECOMMUNICATIONS OY (FI)) 3 September 1998 (1998-09-03) page 6, line 4 -page 8, line 10	1,9,11, 13,27, 29,31, 35,37,39 4,8,16, 19,23, 24,26, 33,36
X A	EP 0 755 141 A (SHARP KK ;SHARP MICROELECT TECH INC (US)) 22 January 1997 (1997-01-22) column 6, line 45 -column 9, line 15 -/--	1,10,11, 13,28, 29,31, 35,38,39 2,6,12, 14,21, 30,40

☒ Further documents are listed in the continuation of box C.☒ Patent family members are listed in annex.

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C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

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X	AL-SANIE A ET AL: "Blind channel estimation and data recovery in DS spread spectrum systems" SIGNAL PROCESSING, AMSTERDAM, NL, vol. 81, no. 8, August 2001 (2001-08), pages 1705-1714, XP004274120 ISSN: 0165-1684	1,11,13, 29,31, 35,39
A	page 1705, right-hand column, paragraph 3 -page 1706, left-hand column, paragraph 2 page 1707, right-hand column, paragraph 5 -page 1708, right-hand column, paragraph 1	2,6,12, 14,21, 30,40

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Patent document cited in search report		Publication date		Patent family member(s)	Publication date
WO 9838772	A	03-09-1998	AU	688228 B1	05-03-1998
			AU	6217598 A	18-09-1998
			CN	1248370 T	22-03-2000
			EP	0963642 A1	15-12-1999
			WO	9838772 A1	03-09-1998
			JP	2001513954 T	04-09-2001
			NO	994072 A	24-08-1999
EP 0755141	A	22-01-1997	EP	0755141 A2	22-01-1997
			JP	9036704 A	07-02-1997

Form PCT/ISA/210 (patent family annex) (July 1992)

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E

ACCURATE AND EFFICIENT SOLUTION OF HANKEL MATRIX SYSTEMS BY FFT AND THE CONJUGATE GRADIENT METHODS

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ABSTRACT

Hankel matrix systems often arise in many problems of signal analysis. Toeplitz matrix systems is a special case of a Hankel matrix equations. Both the auto-correlation and the covariance matrix equations are different forms of the Hankel matrix equations. Trench has developed a direct method that can solve Hankel matrix equations in $O(N^2)$ operations. In this paper, we propose an alternate algorithm. This new method is a combination of the FFT and the conjugate gradient method. The advantage of this new approach is that it is computationally robust to highly ill-conditioned and even singular matrix equations. Preliminary results indicated that for very large complex Toeplitz matrix equations, the CPU time is proportional to N as the number of unknowns as increased, as opposed to N^2 for conventional methods.

1. INTRODUCTION

The objective of this paper is to present a computationally efficient and accurate method for the solution of Hankel matrix equations. The Hankel matrix equations arise in many areas of digital signal processing. Both the covariance and auto-correlation matrix equations are special cases of Hankel matrix equations. The auto correlation matrix is also known as a Toeplitz matrix.

For the problem of interest, let us consider a data sequence $\{x_0, x_1, x_2, x_3, \dots, x_{M+N-1}\}$ of length $M+N$. In many digital signal processing applications, it is required either to make a rational function approximation of the sequence, or to make an all-pole model to fit the data, or perhaps to adaptively find the spectral component in the data. If we choose a filter with coefficients $\{u_0, u_1, u_2, \dots, u_N\}$ then it is well known that the following matrix equation has to be satisfied by the filter coefficients.

$$\begin{bmatrix} x_0 & x_1 & x_2 & \dots & x_N \\ x_1 & x_2 & x_3 & \dots & x_{N+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_N & x_{N+1} & x_{N+2} & \dots & x_{M+N-1} \end{bmatrix} \begin{bmatrix} u_N \\ u_{N-1} \\ \vdots \\ u_1 \\ u_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (1)$$

Symbolically, we can write the above equation

$$[X'] [U'] = [0] \quad (2)$$

where $[X']$ is the well known Hankel matrix. When $M > N$, there are several ways to solve the matrix equation (2) so that one obtains a nontrivial solution for $[U]$.

One way to solve (2) would be to set one of coefficients u_i equal to unity and then transfer that column i of the matrix $[X']$ to the right hand side yielding a rectangular matrix equation. Let us choose for our application $u_N = 1$. Then (2) becomes

$$\begin{bmatrix} x_0 & x_1 & \dots & x_{N-1} \\ x_1 & x_2 & \dots & x_N \\ \vdots & \vdots & \ddots & \vdots \\ x_N & x_{N+1} & \dots & x_{M+N-2} \end{bmatrix} \begin{bmatrix} u_{N-1} \\ u_{N-2} \\ \vdots \\ u_0 \end{bmatrix} = - \begin{bmatrix} x_N \\ x_{N+1} \\ \vdots \\ x_{M+N-1} \end{bmatrix} \quad (3)$$

and in a compact form.

$$[X] [U] = [Y] \quad (4)$$

A possible way to solve (4) then will be to multiply both sides by $[X]^*$, where $*$ denotes the conjugate transpose of the matrix $[X]$, yielding

$$[X]^* [X] [U] = [X]^* [Y] \quad (5)$$

Since $[X]^* [X]$ is a positive definite matrix and (5) can be solved in approximately $\frac{N^3}{6}$ operations by utilizing, for example, Cholesky decomposition. The operation count $\frac{N^3}{6}$ does not include the multiplications necessary to compute $[X]^* [X]$ and $[X]^* [Y]$.

The above solution procedure have the following problems:

- [1] $[X]^* [X]$ is a highly ill-conditioned matrix and can even be singular. Under these circumstances a Cholesky decomposition may not work. This is because truncation and round-off error propagates in the decomposition procedure making the results useless. It is also well known that Hankel matrix equations are notoriously ill-conditioned [1]. For example, Taylor [1] has shown that the condition number of a Hankel matrix is given by the following bound

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The second choice for the solution of (5) is the conjugate gradient method. The conjugate gradient method is an advanced technique for solving the eigenvalue problem. It is based on the fact that the eigenvalues of a Hermitian matrix are real and the eigenvectors are orthogonal. The method is particularly useful for large matrices where the direct method is not feasible. It is a powerful tool for finding the lowest eigenvalue and its corresponding eigenvector. The method is based on the Rayleigh-Ritz method, which is a variational method for finding the minimum of a functional. The conjugate gradient method is a more efficient version of the Rayleigh-Ritz method, which is based on the conjugate gradient method for minimizing a quadratic form. The method is based on the fact that the eigenvalues of a Hermitian matrix are real and the eigenvectors are orthogonal. The method is particularly useful for large matrices where the direct method is not feasible. It is a powerful tool for finding the lowest eigenvalue and its corresponding eigenvector. The method is based on the Rayleigh-Ritz method, which is a variational method for finding the minimum of a functional. The conjugate gradient method is a more efficient version of the Rayleigh-Ritz method, which is based on the conjugate gradient method for minimizing a quadratic form.

$$(71) \quad \theta_{[A]}(x) - [x] = \theta_{[x]}$$

$$(1) \quad {}^0[X][Y] - [X] = [{}^0Y]$$

conjugate gradient method [C]. In this paper, we outline the important features. For the first version, the method minimizes the error between the exact solution and the approximation, whereas for each iteration, where as for the second version the error is minimized. Then one sees

[illegible]

devoted to computation of $[X]^* [X]$ and $[X]^* [Y]$, compared to the solution of (5). More exactly, X^3 operations are required to form $[X]^* [X]$ and $\frac{1}{2} X^3$ is required to solve it. Hence it is desirable to find an alternate method.

where $\text{cond}([X] | [X])$ represents the condition number of the matrix X and is defined as the ratio of the maximum eigenvalue to the minimum eigenvalue of X . A matrix with a large condition number corresponds to an ill-posed problem and very crudely, one may expect that $\log_{10} \{\text{cond}([X] | [X])\}$ is the number of significant digits lost in the solution of (5). Taylor provides a rule of thumb, that approximately N decimal places of accuracy is lost in the solution of (5), where N is the dimension of the matrix X . This is rather significant!

(9) $\gamma \leq \frac{1}{K+1} \{ ([x]_n, [x]) \}$ and $\gamma \leq \frac{1}{K+1} \{ ([x]_n, [x]) \}$

$$b_k = \frac{([X]^* [R]_{k+1})^* ([X]^* [R]_{k+1})}{([X]^* [R]_k)^* ([X]^* [R]_k)} \quad (20)$$

In this case, $[R_k]^* [R_k]$ is reduced at each iteration however, the error in the solution may not decrease monotonically.

The advantages of the two methods are now apparent, namely

i) In solution of (5) we have never explicitly formed $[X]^* [X]$, even though (5) is being actually solved. Hence, we are dealing with the new data and not the processed data.

ii) Since we have not formed $[X]^* [X]$; N^3 operations have never been done. What we needed however is 2 matrix multiplications per iteration i.e. $2N^2$ operations. The solution procedure would be efficient if the number of iterations required is less than $\frac{7N}{12}$. However it has been our experience that convergence is obtained typically in $\frac{N}{3}$ or less iterations.

iii) The two methods still work even when $[X]^* [X]$ is exactly singular, in that case one obtains the minimum norm solution. The rate of convergence in that case is still at least $1/N$.

Many authors [4-8] have applied the conjugate gradient method to signal processing problems. Reference [8] has a large number of references.

3. IMPROVED EFFICIENCY IN COMPUTATION THROUGH THE INTRODUCTION OF FFT

In looking at the expressions (7) - (13) and (14) - (20) of the conjugate gradient methods, it is seen that most of the computation time is used in the evaluation of $[X][P]$ and $[X]^* [R]$. These are two matrix products that are required per iteration. The rest of the computations are simply scalar products. Therefore each of the two matrix multiplications require N^2 operations. We propose to minimize the operation count of the matrix product from N^2 to approximately $2(M+N-1) \log(M+N-1)$ operations by employing FFT.

We observe that the Hankel matrix has a certain specific structure. Therefore the matrix product $[X][P]$

$$[X][P] = \begin{bmatrix} x_0 & x_1 & x_2 \\ x_1 & x_2 & x_3 \\ x_2 & x_3 & x_4 \\ x_3 & x_4 & x_5 \\ x_4 & x_5 & x_6 \end{bmatrix} \cdot \begin{bmatrix} p_2 \\ p_1 \\ p_0 \end{bmatrix} \quad 3 \times 1 \quad (21)$$

can be rewritten as the convolution of two sequences $\{a\}$ and $\{b\}$. Where

$$\{a\} = \{x_0; x_1; x_2; x_3; x_4; x_5; x_6\} \quad (22)$$

$$\{b\} = \{p_0; p_1; p_2; 0; 0; 0; 0\} \quad (23)$$

The result of the matrix product is embedded in the convolution of the two sequences $\{a\}$ and $\{b\}$. This is clear as one observes the 3rd, 4th, 5th, 6th and 7th elements of the convolution of $\{a\}$ and $\{b\}$. We know that convolution can be efficiently carried out by FFT. So if we perform the FFT of the sequences $\{a\}$ and $\{b\}$ and multiply them and then take the inverse FFT, to obtain the sequence $\{c\}$ as

$$\{A\} = \text{FFT} \{a\}$$

$$\{B\} = \text{FFT} \{b\}$$

$$\{C\} = \{a\} * \{b\} = (\text{FFT})^{-1} \{ \text{FFT}(a) \cdot \text{FFT}(b) \} \quad (24)$$

then the desired result is the 3rd to the 7th elements of $\{c\}$. So the total operation counts are:

$$\left. \begin{array}{l} M \cdot N \text{ for matrix product} \\ 2(M+N-1) \log(M+N-1) \text{ using FFT} \end{array} \right\} \quad (25)$$

If $(M+N-1)$ is not a number that is divided by 2, then zero padding could be done to make a number divisible by 2. So the application of the FFT becomes more efficient as the matrix size increases. It is interesting to point out that there is no need to store the matrix $[X]$. All one needs to do is store the $(M+N-1)$ elements of $\{X\}$ in an one dimensional array. The FFT elements of $\{X\}$ are replaced by the transformed sequence. Therefore one needs to take only one Fourier transform in the above discussions instead of two.

We also need to compute $[X]^* [R]$ at each iteration. Again this matrix product can also be performed through the utilization of FFT. Also one could utilize the FFT of the sequence of elements in $[X]$ that are already stored in the array.

Therefore, it is clear that at each iteration one needs to take only 4 FFT's rather than performing $2N^2$ multiplications.

There is also a saving from storage point of view. Instead of requiring the storage of $M \cdot N$ elements of matrix $[X]$ one needs to store only $(M+N-1)$ elements and 4 other auxiliary vectors as required by the conjugate gradient method. This concept of utilizing FFT to evaluate matrix products has also been used before [4, 9, 10].

4. NUMERICAL RESULTS

A sample result of number of unknowns against the CPU time is presented in figure 1. For this figure we consider the solution of a complex

Toeplitz matrix equation $AX = Y$. The matrix is not Hermitian. This type of matrices often arise in electromagnetic signal detection problems. The CPU time against the number of unknowns is presented in figure 1 to obtain a .1% accuracy in the solution. It is seen that as the number of unknowns are increased from 100 to 900 (complex vectors) the CPU time increased only linearly as opposed to N^2 for the conventional methods. It is important to point out that the conventional methods may be quite competitive, typically for $N < 30$ to 50 unknowns.

REFERENCES

- [1] J. M. Taylor, "The condition of Gram matrices and related problems," Proc. Roy. Soc. Edin., Vol. 80A, pp. 45-56, 1978.
- [2] T. K. Sarkar, K. R. Siarkiewicz and R. F. Stratton, "Survey of Numerical Methods for Solution of Large System of Linear Equations for Electromagnetic field problems," IEEE Trans. Antennas & Propagation, Nov. 1981, pp.847-856.
- [3] T. K. Sarkar and E. Arvas, "On a class of finite step iterative methods (conjugate directions) for the solution of an operator equation arising in electromagnetics," IEEE Trans. Antennas & Propagation, Oct. 1985, pp.1058-1066.
- [4] J. R. Fienup, "Phase Retrieval Algorithms: a comparison," Applied Optics, Aug. 1982, pp.2758-2769.
- [5] F. Koehler and M. Taner, "The use of Conjugate gradient algorithm in the computation of predictive deconvolution operators, Geophysics, vol.50, pp.2752-2758, 1985.
- [6] S. Kaneda and O. Nalcioğlu, "Constrained iterative reconstruction by the conjugate gradient method," IEEE Trans. on Medical Imaging, Vol. MI-4, 1985, pp.66-71.
- [7] J. O. Frial, "Adaptive prediction of stationary time series by modified conjugate direction methods," Comm. in statistics, vol. 4, 1975, pp.19-32.
- [8] T. K. Sarkar, "The Conjugate gradient method as applied to Electromagnetic Field Problems," IEEE Antennas & Propagation, Nov. 1981, pp.14-15.
- [9] T. K. Sarkar, E. Arvas and S. M. Rao, "Application of FFT and the conjugate gradient method for the solution of electromagnetic radiation from electrically large and small conducting bodies," IEEE Transactions Antennas and Propagation, May 1986, pp.635-640.
- [10] T. K. Sarkar, S. A. Dianat and B. Z. Mollman, "Deconvolution of impulse response from time to time limited input and output. Theory and Experiment," IEEE Trans. Instrumentation & measurement, Dec. 1985, pp.541-546.

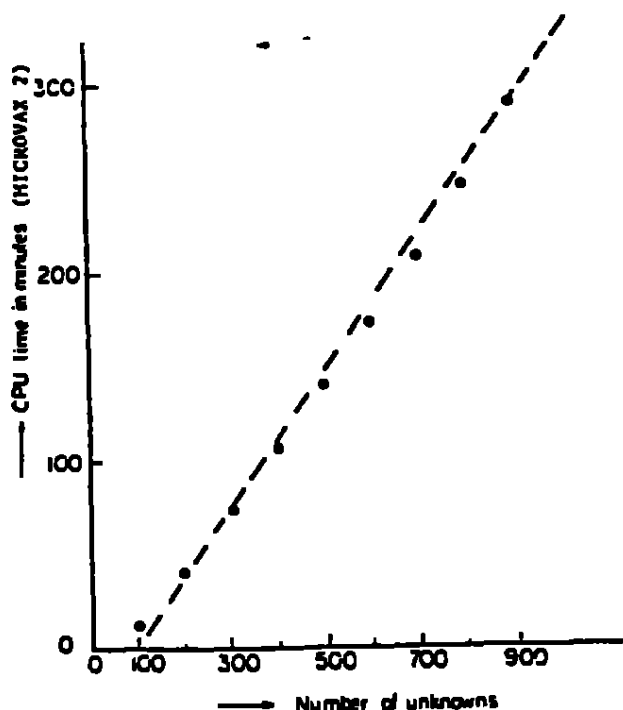


FIGURE : 1

42.12.4

CHANNEL EQUALIZATION WITH ADAPTIVE FILTERING AND THE PRE-CONDITIONED CONJUGATE GRADIENT ALGORITHM

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ABSTRACT

Communication system performance is often degraded by the imperfections of the channel. Additive noise, and actual channel characteristics can be unknown prior to transmission. Use of adaptive equalizers to compensate for these imperfections can significantly improve the performance of the overall system. For highly-correlated received sequences, convergence rate of these equalizers is a strongly limiting factor. This paper presents a novel scheme employing the use of the pre-conditioned conjugate gradient algorithm for channel equalization. Simulation examples are presented for channel equalization problems comparing performance of the LMS algorithm to the pre-conditioned conjugate gradient algorithm for algorithms which minimize the mean square error and the constant-modulus criterion.

1. INTRODUCTION

Adaptive filters can be used to improve performance of communication systems. Unknown channel characteristics can lower bit-error rates and make communication systems unreliable. Employing an adaptive filter to effectively equalize the channel compensates for the finite memory of the channel, and can improve performance in terms of mean-square error and bit-error rate. In Figure 1, a simplified communications channel employing an adaptive equalizer is shown. The details of the transmitter and the receiver are omitted here. For this system, the transmitted sequence is known a priori at the receiver. The use of this brief training sequence allows for channel compensation prior to regular use of the channel for communication. When this training sequence is known, and the delay can be closely approximated, the output error is minimized in a mean square sense.

This output error, $e(n)$, between the adaptive filter output and the training sequence, $d(n)$, can be represented in terms of the received signal, $x(n)$, and the adaptive filter coefficients, $w_i(n)$, by

$$e(n) = d(n) - W^T(n)X(n) \quad (1)$$

where $W(n)$ and $X(n)$ are vectorized versions of the adaptive filter coefficients, and the received signal:

$$W(n) = [w_0(n) \ w_1(n) \ \dots \ w_{N-1}(n)]^T \quad (2)$$

$$X(n) = [x(n) \ x(n-1) \ \dots \ x(n-N+1)]^T \quad (3)$$

Traditional LMS filters attempt to minimize the error signal, $e(n)$ in a mean square sense. The performance function is

$$J(n) = E[e^2(n)]. \quad (4)$$

If the signal $d(n)$ is unknown or other system limitations prevent the re-transmission of the training sequence, then, equalization of the channel via an adaptive filter may be performed by optimizing a slightly different objective function. For many communication systems, some property of the transmitted sequence signal is known at the receiver, particularly the modulus or amplitude. In that case, the adaptation algorithm attempts to minimize the difference between the output of the adaptive filter, $y(n) = W^T(n)X(n)$ and the known modulus. In this situation, the performance function of this constant modulus algorithm (CMA) is

$$J(n) = E[|y(n)|^p - |\delta|^p]^q \quad (5)$$

where δ is the known or expected amplitude (modulus) of the signal and p, q are integers. Treichler et al. [9] applied the LMS algorithm to the case where $p = 1$ and $q = 2$. This algorithm appears to have markedly slower convergence rates for highly colored input channels than traditional mean square error approaches.

Schirsinger [8] presents an experimental survey of a number of adaptation techniques including LMS, transform-domain, lattice filter structures, fast quasi-Newton algorithms and a variant of the conjugate gradient technique presented in [6]. Each of the algorithms presented offered performance improvement over LMS-based algorithms at the expense of increased computational cost. Particularly, the conjugate gradient technique offered the ability for the designer to choose between performance level and computational complexity without switching algorithms. However, this particular variant of the conjugate gradient algorithm leads to fairly high misadjustment or a large excess mean square error [6].

In this paper, pre-conditioners developed by Strang [3], Chan [4], Ku and Kuo [5] are employed in a new quasi-Newton type algorithm to improve the convergence rate and reduce misadjustment. This algorithm allows the designer to choose differing levels of performance based upon desired computational complexity. Simulation examples illustrate that the use of the pre-conditioned conjugate gradient algorithm can significantly improve performance without significantly increasing computation. Section 2 outlines the use of

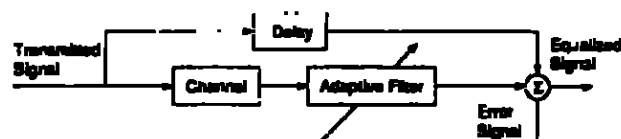


Figure 1. Adaptive filtering application - channel equalization

the pre-conditioned conjugate gradient algorithm is adaptive filtering. Section 3 presents some simulation examples employing both MSE and constant modulus (CM) adaptation algorithms. Finally, Section 4, presents some concluding remarks about performance.

2. PRE-CONDITIONERS IN CONJUGATE GRADIENT ADAPTIVE FILTERING

The conjugate gradient algorithm has been used with success to improve convergence rate [6], [7] and [8]. Buray and Srinath [6], and Schiratzinger [8] use the conjugate gradient algorithm implicitly at every iteration to solve the problem

$$\min_{W(n)} \frac{1}{2} W^T(n) R_{xx}(n) W(n) - P_{xd}^T(n) W(n), \quad (6)$$

where the matrix $R_{xx}(n)$ is the toeplitz-symmetric autocorrelation matrix of the input, and the $P_{xd}(n)$ is the cross-correlation of the received sequence and the training sequence. Use of the conjugate gradient algorithm to solve this minimization problem yields the Wiener-Hopf solution [1]:

$$W_{opt}(n) = R_{xx}^{-1}(n) P_{xd}(n) \quad (7)$$

Use of this method requires a higher-order gradient update expression which raises the computational complexity. In addition, this particular approach was shown to have a larger misadjustment than the LMS algorithm [6]. In this paper, the conjugate gradient algorithm is used directly to calculate the Kalman gain vector [7]. $R_{xx}^{-1}(n)X(n)$ is the quasi-Newton algorithm. The quasi-Newton method for adapting the filter coefficients where the training sequence is known is given by [10]:

$$W(n+1) = W(n) + \mu(n)e(n)R_{xx}^{-1}(n)X(n) \quad (8)$$

Alternatively, the adaptation of the filter coefficients for the constant modulus algorithm may be represented by equation (8) where the error signal, $e(n)$, is replaced by the signal $c(n)$ which is given by [8]:

$$c(n) = (|y(n)|^2 - |\delta|^2)y(n) \quad (9)$$

The Newton method has been shown to dramatically improve convergence rates when the input is highly correlated. Both adaptation algorithms are summarized in Table 1.

The use of the Levinson algorithm to compute the Kalman gain in Marshall, et al. [10] has been shown to dramatically improve convergence performance without significantly increasing computation. However, the autocorrelation matrix is only inverted every N iterations, which can degrade performance for some situations.

By employing the conjugate gradient method to find the Kalman gain vector, a low complexity algorithm may be derived which allows the designer to choose the desired performance level of the adaptive filter. The conjugate gradient algorithm iterates to the solution of the equation (6) except that $X(n)$ replaces $P_{xd}(n)$ and $K(n)$ replaces $W(n)$. The solution to the minimization problem then becomes $K(n) = R_{xx}^{-1}(n)X(n)$.

The conjugate gradient method functions by iteratively constructing N R_{xx} -orthogonal linearly independent vectors, and forming a solution based upon these linearly independent basis vectors. Equivalently, the optimal solution may be represented as:

$$W_{opt}(n) = \alpha_0 D_0 + \dots + \alpha_{N-1} D_{N-1} \quad (10)$$

Table 1. Adaptation Algorithms

Initialization:
$W_i = 0, i = 0, \dots, N-1$
if CMA, $W_{ref} = 1, ref = \text{arbitrary}$
$r(i) = 0, i = 1, \dots, N-1$
$r(0) = 1$
Vector and Matrix Definitions:
$A = \text{diag}\{10^{1/2} \dots 10^{N-1/2}\}$
$R(n) = \{r(0), r(1), \dots, r(N-1)\}^T$
$R_{xx}(n) = \text{toeplitz}\{R(n)\}$
Recursion:
$y(n) = W^T(n)X(n)$
$\epsilon(n) = (y(n) ^2 - 1)y(n)$
$e(n) = d(n) - y(n)$
$R(n) = \alpha R(n-1) + x(n)A X(n)$
$K(n) = R_{xx}^{-1}(n)X(n)$ - via the PCG method
$\mu(n) = \frac{1}{4(d + X^T(n)R_{xx}^{-1}(n)X(n))}$
Tap Weight Updates:
$W(n+1) = W(n) - \mu(n)\epsilon(n)K(n)$ - CMA Algorithm
$W(n+1) = W(n) + \mu(n)e(n)K(n)$ - LMS Algorithm

where the D_i 's represent the conjugate direction vectors. A close approximation to the solution of the Kalman gain may be formed by using M fewer than N of the direction vectors. In this way, the adaptive filtering algorithm may choose performance level, i.e. misadjustment and convergence rate by choosing the number of conjugate direction vectors. Alternatively, a stopping criteria may be defined by comparing the gradient of the performance function to some predefined tolerance value [7].

The rate of convergence of the conjugate gradient algorithm is directly related to the eigenvalue spread of the matrix R_{xx} . The convergence can be accelerated by applying a pre-conditioning step at each iteration of the conjugate gradient algorithm. Instead of solving the system, $R_{xx}K(n) = X(n)$, the pre-conditioned system is solved:

$$\tilde{R}_{xx}\tilde{K}(n) = \tilde{X}(n) \quad (11)$$

where $\tilde{R}_{xx} = P^{-1}R_{xx}P^{-1}$, $\tilde{K}(n) = PK(n)$, and $\tilde{X}(n) = P^{-1}X(n)$. Use of the pre-conditioned CG method requires that the matrix P^{-1} be computed efficiently. For this reason, the matrix P is generally chosen to be circulant, so that a FFT may be used to efficiently compute the inverse.

The convergence rate of the pre-conditioned CG method is governed by the the eigenvalue spread of $P^{-1}\tilde{R}_{xx}$. The lower the condition number of this matrix, the faster the convergence rate. In this paper, the performance of three different forms of the matrix P are compared. The pre-conditioner developed by Strang [3] uses the central half of the diagonals of the original matrix R_{xx} . Strang [3] showed that the eigenvalues of the matrix $P^{-1}\tilde{R}_{xx}$ are clustered around one except for a finite number of outliers for a large class of matrices known as Wiener class matrices. This implies that the PCG method will converge to the optimum solution in fewer than N iterations for this class [2].

The pre-conditioner developed by Chan [4] minimizes the Frobenius norm, $\|P - R_{xx}(n)\|_F^2$, between the original matrix $R_{xx}(n)$ and all circulant matrices P . This solution

Table 2. Pre-Conditioned CG Algorithm

Initialization:
$K_0 = 0$
$P_0 = G_0 = X - R_{xx} K_0$
$\beta_1 = 0$
Step 1: Pre-conditioning - Solve for x_{j-1}
$PZ_{j-1} = G_{j-1}$
Step 2: CG Iteration - Compute
$\beta_j = \frac{x_{j-1}^T G_{j-1}}{x_{j-1}^T G_{j-1}}$
$D_j = x_{j-1} + \beta_j D_{j-1}$
$\alpha_j = \frac{x_{j-1}^T G_{j-1}}{D_j^T R_{xx} D_j}$
$K_j = K_{j-1} + \alpha_j D_j$
$G_j = G_{j-1} - \alpha_j R_{xx} D_j$
Step 3: if $\ G_j\ ^2 > TOL$, or $j < M \leq N$ return to Step 1

to this minimization has a closed form expression, so the pre-conditioner may be easily computed.

Ku and Kuo [5] developed a broad class of pre-conditioning matrices which attempt to use all of the original elements of matrix $R_{xx}(n)$. The pre-conditioners developed by Chan and Strang are special cases of two of the four classes of pre-conditioners developed by Ku and Kuo. One case of the first class of pre-conditioner developed by Ku and Kuo is included for comparison. The pre-conditioned conjugate gradient algorithm is summarized in Table 2.

3. EXPERIMENTAL RESULTS

In the first equalization experiment, the adaptive filter is attempting to model the inverse of the channel which has the impulse response [1]:

$$h(n) = \delta(n) + 0.3\delta(n-2) \quad (12)$$

This channel mildly colors the transmitted sequence. The eigenvalue spread of the transmitted BPSK sequence of equally likely ± 1 's was changed from 1 to 3.07 by the channel.

For the LMS-constant modulus algorithm, the parameter μ was set to $1/25$. For the PCG-Newton method, α was set to 0.95, and the normalizing factor, δ in the self-adjusting μ factor was set to 0.05. The additive noise floor was fixed at -100 dB. The resulting mean square error and the reconstructed modulus of the signal conjugate gradient algorithm are plotted in Figure 2. The conjugate gradient algorithm was limited to three iterations ($M = 3$) in both the CMA and MSE algorithms. The tolerance value or stopping condition chosen for the gradient expression was 10^{-4} . For this example, the LMS algorithm appears to exhibit convergence and misadjustment performance similar to the three preconditioned conjugate gradient algorithms employing preconditioners by Strang [3], Chan [4], and Ku et al. [5]. In Figure 3, the results of the MSE algorithm appear to illustrate that the pre-conditioners appear to have little impact in either changing the misadjustment or the convergence rate of the adaptive filter. Interestingly, the excess mean square error for the MSE algorithm is higher than the misadjustment for the CMA algorithm.

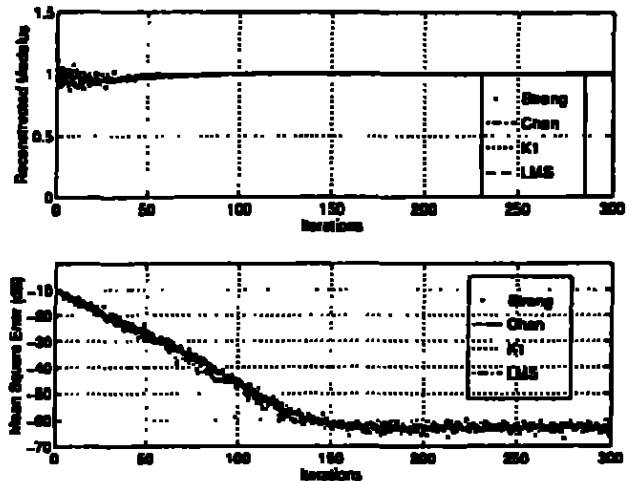


Figure 2. Channel A - CMA Algorithm - Reconstructed Modulus and MSE

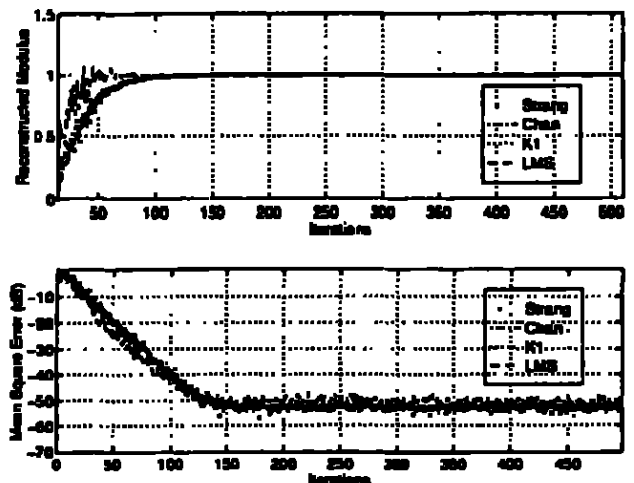


Figure 3. Channel A - MSE Algorithm - Reconstructed Modulus and MSE

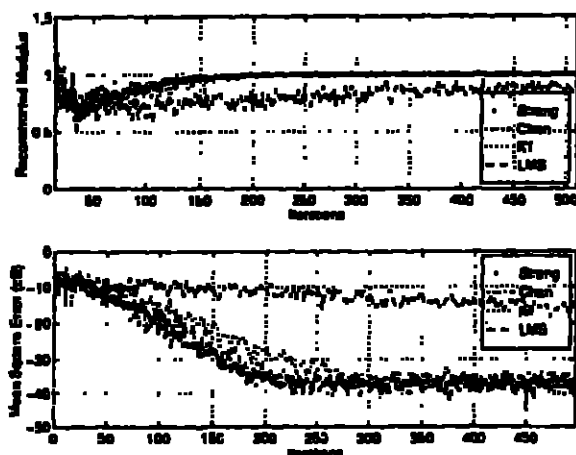


Figure 4. Channel B - CMA Algorithm - Reconstructed Modulus and MSE

In the second equalization experiment, the adaptive filter models a channel which highly correlates the data and has a resulting eigenvalue spread of 100. The model for the channel is given by [1]

$$h(n) = \frac{1}{2}[1 + \cos(2\pi(i-2)/1V)], \quad i = 1, 2, 3 \quad (13)$$

where W was set to 3.5. For this algorithm, μ was set to 1/50 for the CMA algorithm and 1/20 for the MSE algorithm. The parameters for the PCG-Newton method remained unchanged.

Figure 4 appears to show that the use of PCG-Newton method is strongly warranted in the CMA algorithm. The LMS algorithm exhibits a fairly slow convergence rate, and a fairly high misadjustment which is not displayed in order to illustrate the different convergence behaviors of the three different pre-conditioners. The filter converges to around -30 db which is about 10 db higher than the PCG-Newton algorithm. The difference in convergence among differing versions of the PCG-Newton method are small. Chan's preconditioner lags slightly behind that of the Strang's and Ku and Ku's. The difference appears to be only a few iterations relative to the hundreds of additional iterations required by LMS to reach a higher overall misadjustment level. Figure 5 appears to again demonstrate that the MSE performance of the PCG-Newton based algorithm is superior to the LMS based algorithm. The MSE algorithm does however appear to converge more rapidly than the CMA algorithm for this same model. However, the LMS algorithm still requires several hundred additional iterations over any of the pre-conditioner based algorithms.

4. CONCLUDING REMARKS

The pre-conditioned conjugate gradient algorithm for channel equalization was observed to greatly enhance convergence and misadjustment performance over traditional LMS-based mean square error and constant modulus algorithms. The use of pre-conditioners offers the designer a choice of performance level by choosing the stopping condition of the conjugate gradient algorithm either by tolerance or a fixed number of iterations. By choosing a relatively small number of fixed iterations of the algorithm, the computational complexity of the pre-conditioned CG method

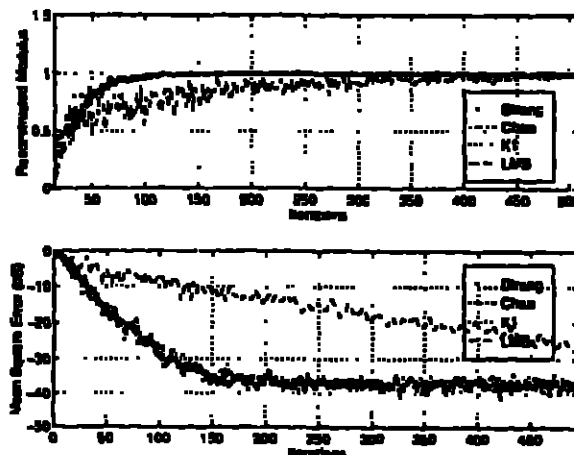


Figure 5. Channel B - MSE Algorithm - Reconstructed Modulus and MSE

was observed to be $O(NM \log(N))$ where M is the number of iterations of the inner loop, and N is the order of the adaptive filter. The constant modulus algorithm that traditionally suffers from poor performance relative to the MSE algorithm appeared to have its convergence rate made comparable to traditional MSE algorithm for channels which highly correlate data.

REFERENCES

- [1] S. Haykin, *Adaptive Filter Theory*. Englewood Cliffs, NJ: Prentice-Hall, 1986.
- [2] D. Luenberger, *Linear and Non-Linear Programming*. Reading, MA: Addison-Wesley, 1984.
- [3] R. Chan and G. Strang, "Toeplitz equations by conjugate gradients with circulant preconditioner," *SIAM Journal Sci. Stat. Comput.* vol. 10, no. 1, pp. 104-119, Jan., 1989.
- [4] T. Chan, "An optimal circulant preconditioner for toeplitz systems," *SIAM Journal Sci. Stat. Comput.*, vol. 9, no. 4, pp. 766-771, Jul. 1988.
- [5] T. Ku and C. Ku, "Design and analysis of toeplitz preconditioners," *IEEE Trans. Signal Process.*, vol. 40, no. 1, pp. 129-141, Jan. 1992.
- [6] G. Boray and M. Srinath, "Conjugate gradient techniques for adaptive filtering," *IEEE Trans. Circuits and Systems*, vol. 39, no. 1, pp. 1-10, Jan. 1992.
- [7] A. Hull, "Orthogonalization techniques for adaptive filters," Ph. D. dissertation, University of Illinois at Urbana-Champaign, Urbana, IL, 1993.
- [8] T. Schirring, "Designing adaptive equalizers based on the constant modulus error criterion," M.S. thesis, University of Illinois at Urbana-Champaign, Urbana, IL, 1994.
- [9] J. Treichler, C. Johnson, and M. Larimore, *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1986.
- [10] D. Marshall and W. Jenkins, "A fast quad-Newton adaptive filtering algorithm," *IEEE Trans. Signal Proc.* vol. 40, no. 7, pp. 1652-1662, Jul. 1992.

CONJUGATE GRADIENT BASED MULTICHANNEL DECISION FEEDBACK EQUALIZATION FOR DIGITAL TELEVISION

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ABSTRACT

In this paper, we examine the performance of a multichannel Decision Feedback Equalizer (DFE) employing Conjugate Gradients (CG) based adaptive filtering in the context of Digital Television based on the 8-VSB standard. The 8-VSB modulation scheme employs real-valued 8-PAM symbols but complex root-raised cosine (spectral) pulse shaping for bandwidth efficiency. Thus, even in the case of a single receive antenna, two virtual channels may be realized via the real and imaginary parts of the received signal. A dual virtual channel minimum mean-square error (MMSE) DFE is derived. Given multiple channels, the high symbol rate of 8-VSB based TV, 10.76 MHz, and the rather large delay spreads one has to contend with in the UHF and VHF bands, the composite DFE weight lies in a space of high dimensionality. We thus implement the MMSE-DFE adaptively using the conjugate gradient (CG) algorithm since it has been shown to effect reduced-rank adaptive filtering, for improved performance with insufficient training, and is also amenable to "smart" initialization.

1. INTRODUCTION

The terrestrial broadcast standard for digital television in the United States is based on 8-level vestigial sideband modulation (8-VSB) [1]. According to the standard, a field sync segment is transmitted every 24.2ms containing 704 known symbols — four segment sync symbols, a length 511 pseudo-noise (PN) sequence, and three length 63 PN sequences. These symbols may be used to train an adaptive equalizer. Since the broadcast environment is characterized by long channel impulse responses [2], long equalizers are required, which makes it more difficult for an adaptive equalizer to converge within the available training period.

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In order to use the training sequence, a correlation must be performed to locate the training sequence within a frame. This correlation operation may be used to estimate the multipath channel and the channel estimate may then be used to aid the convergence of a training-based adaptive equalizer. More sophisticated techniques may also be applied to improve the channel estimate Özen et al [2].

Recent work on reduced-rank filtering [3] has linked the multi-stage nested Wiener filter to the conjugate gradient (CG) algorithm. Chang and Willson [4] have analyzed adaptive implementations of the CG algorithm. Chowdhury and Zoltowski [5] have successfully applied the CG algorithm to DS-SS feedforward equalizers and found that it converges more quickly than the recursive least squares (RLS) algorithm with reduced computational complexity. In this paper, we show how the convergence time of decision feedback equalizers (DFE's) for 8-VSB based on the CG algorithm can be considerably improved through initialization based on a channel estimate. We begin by describing the 8-VSB system model and deriving expressions for real and complex minimum mean-square error (MMSE) equalizers. These equalizers are then implemented adaptively using the CG, RLS, and least mean squares (LMS) algorithms. We compare several methods of initialization to determine how each affects convergence and then apply the best methods to initialize equalizers using channel estimates.

2. SYSTEM MODEL

The baseband equivalent discrete-time system model is shown in Fig. 1 where symbol-rate sampling is assumed. The transmitted symbol sequence is represented by $s[k]$ where, in this study, we assume that the symbols are independent and equally probable. Symbols in the 8-VSB constellation are taken from the set $\{-7, -5, -3, -1, 1, 3, 5, 7\}$. The continuous-time channel contains M paths with delays τ_i and complex gains α_i . We assume that the path with maxi-

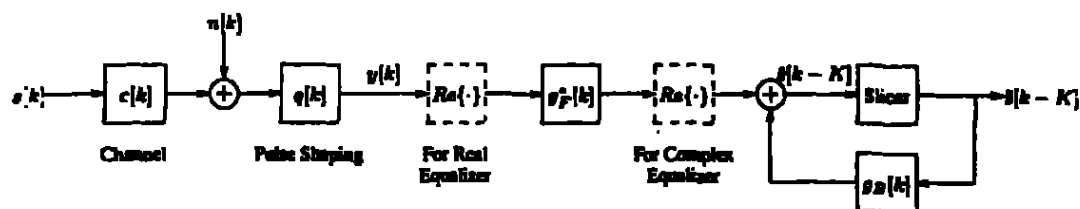


Fig. 1. System model.

imum gain magnitude occurs at a delay of zero. When convolved with the complex transmitted pulse $q(t)$, the channel $c(t)$ becomes $c(t) = \sum_{i=1}^M \alpha_i q(t - \tau_i)$. In the ATSC standard[1], the pulse $q(t)$ is a square-root raised cosine pulse expressible as $q(t) = e^{j\pi F_s t} p_{RRC}(t)$, where $F_s = \frac{1}{T_s}$ is the symbol rate and $p_{RRC}(t)$ is a square-root raised cosine pulse with zero crossings at non-zero multiples of $2T_s$. The real part of $q(t) + q^*(-t)$ is a raised cosine pulse with zero crossings at non-zero multiples of T_s .

The discrete-time channel is found by sampling $c(t)$ at the symbol rate: $c[k] = c(kT_s)$. The noise sequence $n[k]$ is complex additive white Gaussian noise with a power spectral density of N_0 . The receiver filter $q[k]$ is matched to the transmitter pulse shaping filter so that $q[k] = q^*(-kT_s) = q^*(kT_s)$, where the asterisk superscript (*) denotes complex conjugation. The composite channel at the input to the equalizer is defined to be $h[k] = c[k] * q[k]$ (here, the large asterisk denotes convolution) and therefore $y[k] = s[k] * h[k] + v[k]$, where $v[k] = n[k] * q[k]$.

3. EQUALIZER DERIVATIONS

As shown in Fig. 1, $y[k]$ may be equalized using either a complex or real feedforward section $g_F[k]$. For the complex equalizer, the focus of this study, the real part of the signal is taken after the feedforward section. As will be evident in the derivation to follow, the complex equalizer may be viewed as a dual virtual channel equalizer. The composite channel $h[k]$ is assumed to have length $L_h + 1$ with L_{ha} taps prior to the peak of the maximum magnitude path and L_{hc} taps afterward; thus, $L_h = L_{ha} + L_{hc}$. We represent the channel by a vector

$$h = [h[-L_{ha}], \dots, h[-1], h[0], h[1], \dots, h[L_{hc}]]^T. \quad (1)$$

We also use a finite length representation of the receiver matched filter $q[k]$ with length $2L_q + 1$:

$$q = [q[-L_q], \dots, q[-1], q[0], q[1], \dots, q[L_q]]^T. \quad (2)$$

The equalizer is assumed to have $N_F + 1$ feedforward taps and N_B feedback taps. Assembling $N_F + 1$ consecutive

samples of $y[k]$ into a vector y yields

$$y[k] = Hs[k] + Qn[k], \quad (3)$$

where $n[k] = [n[k + L_q] \dots n[k - L_q - N_F]]^T$, $s[k] = [s[k + L_{ha}] \dots s[k - L_{hc} - N_F]]^T$, and H^T and Q^T are convolution matrices of commensurate dimensions. The symbol estimate $\hat{s}[k - K]$ is given by

$$\hat{s}[k - K] = \text{Re}\{g_F^H y[k]\} + g_B^T s_B[k - K - 1] \quad (4)$$

where K is the cursor location, $g_F = [g_F[0], \dots, g_F[N_F]]^T$, $g_B = [g_B[1], \dots, g_B[N_B]]^T$, $s_B[k] = [s[k], \dots, s[k + 1 - N_B]]^T$ and the superscript H denotes conjugate transpose. In this analysis we assume that all decisions are correct. If we adopt the convention that the subscripts R and I indicate the real and imaginary parts of quantities, then

$$\begin{aligned} \hat{s}[k - K] &= g_{FR}^T y_R[k] + g_{FI}^T y_I[k] + g_B^T s_B[k - K - 1] \\ &= g_{FC}^T y_C[k] + g_B^T s_B[k - K - 1], \end{aligned} \quad (5)$$

where $g_{FC} = [g_{FR}^T \ g_{FI}^T]^T$ and $y_C[k] = [y_R^T[k] \ y_I^T[k]]^T$. We may now write

$$y_C[k] = H_C s[k] + Q_C n_C[k], \quad (6)$$

where $H_C = [H_R^T \ H_I^T]^T$, $Q_C = \begin{bmatrix} Q_R & -Q_I \\ Q_I & Q_R \end{bmatrix}$, and $n_C[k] = [n_R^T[k] \ n_I^T[k]]^T$.

The equalizer is found by minimizing the mean-square error (MSE) yielding the Wiener-Hopf equations

$$\begin{bmatrix} R_{y_C y_C} & R_{y_C s_B} \\ R_{y_C s_B} & R_{s_B s_B} \end{bmatrix} \begin{bmatrix} g_{FC} \\ g_B \end{bmatrix} = \begin{bmatrix} r_{y_C} \\ r_{s_B} \end{bmatrix}, \quad (7)$$

where $R_{s_B s_B} = E_s I_{N_B}$ and $r_{s_B} = 0$. The solution may be expressed as

$$g_{FC} = \left(R_{y_C y_C} - \frac{1}{E_s} R_{y_C s_B} R_{y_C s_B}^T \right)^{-1} r_{y_C} \quad (8)$$

$$g_B = -\frac{1}{E_s} R_{y_C s_B}^T g_{FC}. \quad (9)$$

where

$$\tau_{syc} = E\{s[k-K]v_C[k]\} = \varepsilon_s H_C \delta_{K+L_{hc}} \quad (10)$$

$$R_{v_C v_C} = E\{v_C[k]v_C^T[k]\} = \varepsilon_s H_C H_C^T + N_0 Q_C Q_C^T$$

$$R_{v_C s_B} = E\{v_C[k]s_B^T[k-K-1]\} = \varepsilon_s H_C \Delta_K. \quad (11)$$

Here we have used the fact that $R_{s_B s_B} = E\{s_B[k]s_B^T[k]\} = \varepsilon_s I_{N_B}$ and $\tau_{s_B s_B} = E\{s[k-K]s_B[k]\} = 0$. The vector $\delta_{K+L_{hc}}$ contains all zeros except for a one in element $K+L_{hc}+1$. The matrix Δ_K is defined by

$$\Delta_K = \begin{bmatrix} 0_{(L_{hc}+K+1) \times N_B} \\ I_{N_B} \\ 0_{(N_F+L_{hc}-K-N_B) \times N_B} \end{bmatrix} \quad (12)$$

when $0 \leq K \leq L_{hc} + N_F - N_B$.

4. ADAPTIVE ALGORITHMS

4.1. Conjugate gradient algorithm

Since the conjugate gradient algorithm solves equations of the form $Rg = r$, the algorithm requires estimates of R and r . Since the channels were static in this particular simulation study, we estimated the components of R and r in Eqs. 7 using a running average. That is, to estimate $R_{uv} = E\{u[n]v^T[n]\}$ at time k , we use the expression

$$R_{uv}[k] = \frac{1}{k+1} (kR_{uv}[k-1] + u[k]v^T[k]). \quad (13)$$

The adaptive CG algorithm used is listed in Algorithm 1; note the use of the Hestenes-Stiefel formula for the calculation of the conjugate direction step size β_k [6].

The initial values $R_{v_C v_C,0}$, $R_{v_C s_B,0}$, and $\tau_{syc,0}$ in the first step of the algorithm may be zero matrices. However, if an estimate of the channel h is available, it may be used to initialize all the matrices comprising the Wiener-Hopf equations as per Eqs. 10-11 in order to accelerate convergence rather dramatically. As stated previously, a running correlation needs to be performed to locate the training sequence; that in and of itself can provide an initial channel estimate. Channel based initialization of the matrices is accommodated in the running average approach signified by Eq. 13 via the modified averaging

$$R_{uv}[k] = \frac{1}{k+k_w+1} ((k+k_w)R_{uv}[k-1] + u[k]v^T[k]), \quad (14)$$

where k_w is the initial weighting of $R_{uv,0}$.

5. SIMULATIONS

The equalizers were applied to four channels with $M = 4$ paths in each channel. The gains of each path were generated randomly. The maximum gain was normalized to one

Algorithm 1 Adaptive Conjugate Gradient Algorithm

```

 $R_{v_C v_C}[0] = R_{v_C v_C,0}$ ,  $R_{v_C s_B}[0] = R_{v_C s_B,0}$ ,  $\tau_{syc}[0] = \tau_{syc,0}$ .
 $R_{s_B s_B}[0] = \varepsilon_s I_{N_B}$ ,  $\tau_{s_B s_B}[0] = 0$ 
 $g[0] = g_0$ ,  $t[0] = 0$ 
for  $k = 1, \dots$  do
    Update  $R_{v_C v_C}[k]$ ,  $R_{v_C s_B}[k]$ ,  $R_{s_B s_B}[k]$ ,  $\tau_{syc}[k]$ , and  $\tau_{s_B s_B}[k]$ 
     $t[k] = R[k]g[k-1] - r[k]$ 
    If  $k = 1$  or  $k \bmod k_0 = 1$  then
         $b[k] = -t[k]$ 
    else
         $\beta_k = \frac{(t[k] - t[k-1])^T t[k]}{(t[k] - t[k-1])^T p[k-1]}$ 
         $b[k] = -t[k] + \beta_k b[k-1]$ 
    end if
     $\alpha_k = \frac{b^T[k]t[k]}{b^T[k]R[k]b[k]}$ 
     $g[k] = g[k-1] + \alpha_k b[k]$ 
end for

```

and located at zero delay. Three random delays were generated via independent uniform random variables across a delay spread of -30 to 250 symbols, relative to zero delay, and the remaining normalized gains were assigned to these paths. Channel 1 had ghosts at 19.4, 176.7, and 228.1 symbol times with gains of -6.45 , -0.97 , and -0.28 dB, respectively. Channel 2 had ghosts at -13.8 , 84.9 , and 220.2 symbol times with respective gains of -7.98 , -2.39 , and -5.59 dB. Channel 3 had ghosts at -27.2 , 68.8 , and -4.97 197.7 symbol times with respective gains of -13.86 , -4.97 , and -4.67 dB. Channel 3 had ghosts at 8.9 , 25.6 , and 26.8 symbol times with respective gains of -8.33 , -4.99 , and -1.67 dB. The complex pulse shape at the transmitter and receiver had an excess bandwidth of 0.1152 [1] and was modeled using 61 samples centered at the main tap (i.e., $L_q = 30$). The equalizers had 101 feedforward taps and 300 feedback taps and the cursor was positioned at tap 76 in the feedforward section.

All simulations were performed at a received SNR of 30dB, where the SNR is defined at the input to the receiver matched filter. The SNR is calculated as $\text{SNR} = \varepsilon_s \|c\|^2 / N_0$, where $\varepsilon_s = 21$, $c = [c[-L_{hc} + L_q], \dots, c[L_{hc} - L_q]]^T$, and the noise power is based on the system bandwidth.

The LMS and RLS parameters were chosen by simulating various parameter combinations and choosing the parameters which yielded the fastest stable convergence. The LMS step size was $\mu = 10^{-5}$, and the RLS parameters were $\lambda = 0.999$ and $\delta = 10^{-3}$. In the CG algorithm, the conjugate direction was re-initialized after $k_0 = 502$ steps [6].

We compare several methods of initializing the CG algorithm. It is possible to initialize the equalizer tap weights,

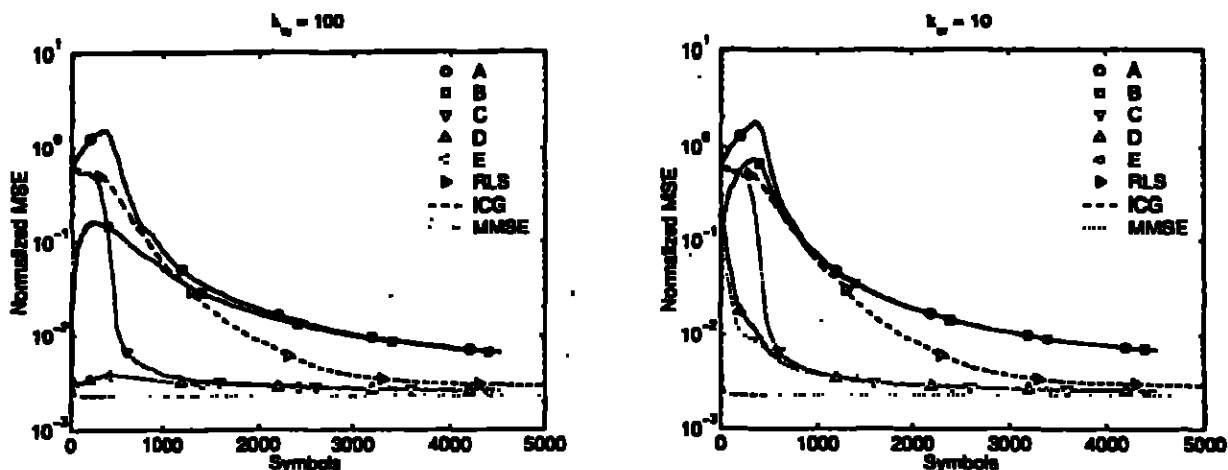


Fig. 2. Convergence of complex CG equalizers initialized using actual channels at a received SNR of 30dB. The plot on the left is for an initial weight of $k_w = 100$. The plot on the right is for an initial weight of $k_w = 10$. The letters refer to the initialization descriptions in the text.

the correlation matrices and vectors, and we may initialize the feedback tap contents to zero or fill the taps with training symbols. When we fill the feedback taps with training symbols, we use the first 300 training symbols, so these symbols are no longer available to train the equalizer. We simulated five different initializations:

- Minimal initialization.** The equalizer weights are initialized to zeros except for a one at the cursor location. The feedback tap delay line contains zeros when training begins. For the CG algorithm, R_{pccr0} , R_{pccr0} , and r_{pccr0} (R_{pccr0} , R_{pccr0} , and r_{pccr0}) are zero matrices; $R_{pccr0}[0]$ and $r_{pccr0}[0]$ are initialized as noted in Algorithm 1.
- Matrix initialization.** In this case, we initialize the correlation matrices using the actual channel h and noise variance N_0 .
- Feedback tap initialization.** Here, we fill the feedback taps with training symbols prior to beginning adaptation. The correlation matrices are not initialized using the channel.
- Matrix and feedback tap initialization.** This is a combination of cases B and C. We initialize the matrices using the actual channel and noise variance, and we fill the feedback taps with training symbols.
- Matrix and feedback tap initialization with equalizer tap weight initialization.** This case is identical to case D except, in addition, we provide a simple initialization of the equalizer tap weights using the negative of the post-cursor portion of the channel.

In addition to these initializations, we also computed an Indirect DFE via the CG based solution [6] of $Rg = r$ and calculated the MSE as a function of the iteration. This could be implemented as an indirect adaptation of the equalizer taps by tracking the channel estimate and calculating the tap weights by forming R and r from the channel estimate. Finally, we reiterate that performing matrix initialization of the RLS algorithm similar to that described in B would be difficult because the matrix updated by the RLS algorithm is the inverse of the correlation matrix.

The results of this simulation for the complex equalizer are shown in Fig. 2 along with the RLS results from the previous simulation. The matrix initialization alone (B) does not yield a large improvement in the convergence time. The MSE initially drops steeply but then rises before falling again. This can be attributed to the zeros initially in the feedback taps which initially direct the matrix averages away from their initial values. Feedback tap initialization (C) does yield a significant improvement in convergence time, but a flat region remains in the MSE. This region can be attributed to the time it takes for the matrix averages to stabilize. As a result, the combination of B and C, shown as D, removes the flat region found in feedback tap initialization and a vast improvement is seen over minimal initialization (A). With an initial weighting of $k_w = 100$, the initial convergence trajectory approaches that of ICG. Even with $k_w = 10$, the normalized MSE quickly reaches 10^{-2} . When the equalizer taps are also initialized (E), there is a slight further improvement in convergence time. Finally, we test our initialization scheme using an estimated channel. We simulated the equalizer using the same four channels, but

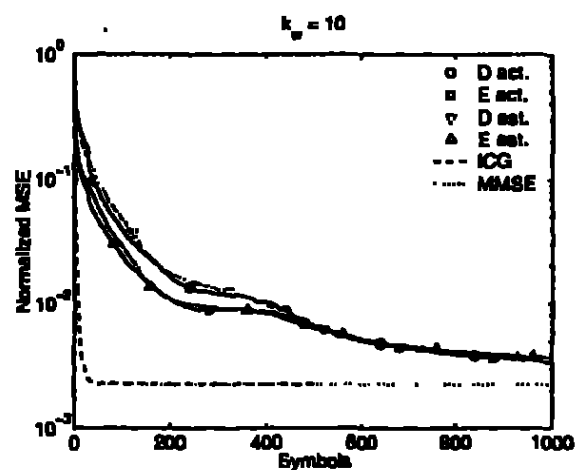
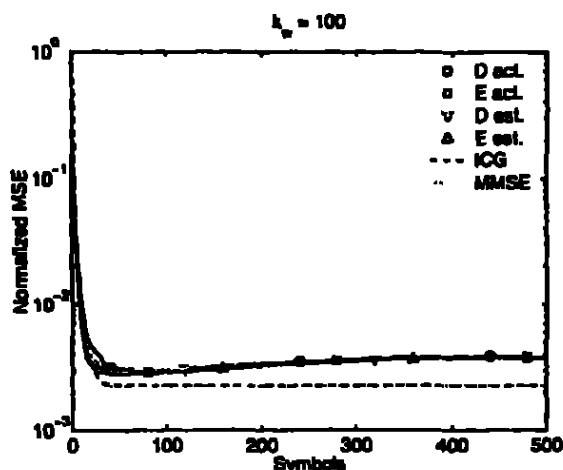


Fig. 3. Convergence of complex CG equalizers initialized using channel estimates at a received SNR of 30dB. The convergence curves for initialization with the actual channels and the estimated channels are compared.

estimated the channel using the scheme described by Özen, et al [2]. The channel estimation is performed in two steps using symbol-spaced received samples prior to the receiver matched filter. In the first step, the received samples are correlated with the stored training sequence and thresholding is applied in order to determine the locations of the multipath delays. The purpose of the second step is to incorporate the transmitted pulse shape $q(t)$ into the channel impulse response. To do this, we locate three copies of $q(t)$ shifted by one-half of a symbol period around each multipath location and estimate complex scaling factors using a modified least squares approach. Further details may be found in [2].

The channel estimate is used to initialize the correlation matrices in the equalizer prior to adaptation and, for case E, to initialize the feedback tap weights. We simulated initialization cases D and E for the complex equalizer and the results are compared with the previous results using the actual channel in Fig. 3. For both $k_w = 10$ and $k_w = 100$, there is little difference between the convergence rates when the equalizers are initialized with the estimated channels and the convergence rates using the actual channels. With $k_w = 100$, the equalizer is able to converge within the available number of training symbols in a single VSB training segment.

6. CONCLUSIONS

Applying adaptive conjugate gradient (CG) to equalization in an 8-VSB system, we found that initializing the correlation matrices while filling up the feedback taps with known training symbols yielded a substantial improvement in convergence time — much more than either initialization scheme

yielded on its own — facilitating convergence within a single VSB training segment.

7. REFERENCES

- [1] Advanced Television Systems Committee, "ATSC standard: Digital television standard, revision B," Document A/53B, 7 August 2001.
- [2] S. Özen, M. D. Zoltowski, and M. Fimoff, "A novel channel estimation method: Blending correlation and least-squares based approaches," in *ICASSP 2002*, Orlando, FL, May 2002, To be published.
- [3] G. Dietl, M. D. Zoltowski, and M. Joham, "Reduced-rank equalization for edge via conjugate gradient implementation of multi-stage nested wiener filter," *2001 Vehicular Technology Conference (Fall)*, Atlantic City, NJ, 7-11 October 2001, vol. 3, pp. 1912-1916.
- [4] Pi Sheng Chang and Alan N. Willson, Jr., "Analysis of conjugate gradient algorithms for adaptive filtering," *IEEE Transactions on Signal Processing*, vol. 48, no. 2, pp. 409-418, February 2000.
- [5] Samina Chowdhury and Michael D. Zoltowski, "Application of conjugate gradient methods in MMSE equalization for the forward link of DS-CDMA," *2001 Vehicular Technology Conference (Fall)*, Atlantic City, NJ, 7-11 Oct. 2001, vol. 4, pp. 2434-2438.
- [6] Edwin K. P. Chong and Stanislaw Zak, *An Introduction to Optimization*, Wiley, New York, NY, 1996.

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ESTIMADOR DE CANAL CONJUGADO Y EFICIENTE BASADO EN GRADACIONES

Campo Técnico del Invento

El presente invento se relaciona con un estimador de canal conjugado y eficiente basado en gradaciones.

Antecedentes del invento

Desde que se adopto el estándar de televisión digital_(DTV)ATSC en 1996 en los Estado Unidos, ha habido un esfuerzo continuo para mejorar el diseño de los receptores construidos para la señal ATSC DTV. El obstáculo principal al que se enfrentan los diseñadores diseñando receptores que logren una buena recepción es la presencia de interferencias multipath en el canal.

El canal de emisión de televisión es un ambiente multipath relativamente severo de debido a la variedad de condiciones que se encuentran en el canal y en el receptor. Los canales están caracterizados por respuestas a impulsos que pueden tener un largo de varios cientos de símbolos, de tal manera que señales con una interferencia fuerte pueden llegar al receptor antes y después de la señal con una amplitud mayor. Adicionalmente, la señal que se trasmite a través del canal está sujeta a condiciones en el canal que varían con el tiempo debido al movimiento del transmisor y de los reflectores de la señal, vibraciones de avión, y, para recepción interior, personas caminando alrededor del cuarto. Si la recepción móvil se desea, el movimiento del receptor también se debe considerar. Aún más, la señal ATSC DTV utiliza una banda rudimentaria codificada trellis de 8-niveles (generalmente referida como 8T-VSB o, sencillamente, como 8-VSB) como método de modulación. Los símbolos de datos 8-VSB son reales y tienen una forma de pulsación de la señal que es compleja. Únicamente la parte real de la forma compleja de la pulsación es una pulsación Nyquist. Por lo tanto, incluso si no hay camino

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multiple(multipath), la parte imaginaria de la forma de la pulsación compleja contribuye a la interferencia inter símbolo (ISI) cuando el incremento en el canal visto por el ecualizador no es real.

La interferencia multipath y de inter símbolo afecta en forma adversa la habilidad del receptor para recibir correctamente los símbolos transmitidos por el transmisor. Por lo tanto, los diseñadores añaden ecualizadores a los receptores para cancelar el efecto de la interferencia de camino multiple(Multipath) y de inter símbolo y por lo tanto mejorar la recepción de la señal.

Porque el canal no se conoce *a priori* en el receptor, el ecualizador debe poder modificar la respuesta para corresponderse con las condiciones del canal que encuentra y adaptarse a los cambios en las condiciones de esos canales. Para ayudar en la convergencia de un ecualizador adaptable a las condiciones del canal, el segmento de campo sync del marco como se define en el estándar ATSC se puede utilizar como una secuencia de entrenamiento para el ecualizador. Pero cuando la ecualización se hace en el tiempo de dominio, se requieren ecualizadores largos (aquellos con muchos reguladores) debido a los impulsos de respuesta del canal largo que caracteriza al canal.

El receptor Grand Alliance original utilizaba un ecualizador adaptable de retroalimentación de decisión (DFE) con 256 reguladores. Este ecualizador adaptable de retroalimentación de decisiones adaptó el canal utilizando un algoritmo estándar least mean square (LMS), y se entrenó con el segmento de campo sync del cuadro transmitido. El algoritmo LMS converge lentamente, incluso con los 256 reguladores, no siempre convergen durante una secuencia única de entrenamiento. Porque el segmento de campo sync se trasmite relativamente con poca frecuencia (alrededor de 260.000

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símbolos), el tiempo total de convergencia para este ecualizador es bastante largo si el ecualizador únicamente se adapta con símbolos de entrenamiento antes de la convergencia.

Por lo tanto, para adaptar los ecualizadores a las variaciones del canal que ocurren entre secuencias de entrenamiento, se ha sugerido la adición de métodos ciegos y dirigidos a las decisiones a los ecualizadores. Sin embargo, cuando se incrementan en un sistema realista, estos métodos pueden requerir varios campos de datos para lograr la convergencia, y la convergencia puede no lograrse bajo condiciones de caminos múltiples difíciles.

De cualquier forma, porque las señales de caminos múltiples (multipath) en el canal de emisión pueden llegar con muchos símbolos luego de la señal principal, el ecualizador de retroalimentación de decisión se utiliza en forma invariable en las aplicaciones 8-VSB.

Se ha discutido que se requiere un ecualizador de retroalimentación de decisión debido al aumento en el error mean square (MSE) entre secuencias de entrenamiento. Sin embargo, la adaptación del ecualizador entrenado en la simulación que apoya este argumento se congeló entre secuencias de entrenamiento. Es posible que un ecualizador dirigido hacia las decisiones con una buena ejecución de seguimiento pueda seguir las variaciones en el canal probado.

Se han propuesto algoritmos ciegos basados en el algoritmo Sato y en el algoritmo Godard de la constante de modulo (CMA). El término de error en ambos de estos algoritmos utilizan la unión continua del término dirigido hacia las decisiones con el término ciego. Esta combinación permite una transición suave entre la modalidad ciega y la modalidad dirigida hacia las decisiones.

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Sin embargo, cuando se incrementa en un sistema realista, estos algoritmos también pueden tomar varios campos de datos para converger.

Tal como se mencionó anteriormente, los ecualizadores adaptables que utilizan el algoritmo least mean square (LMS) pueden converger lentamente o no converger dependiendo de las condiciones del canal. De convergencia se puede afectar adversamente si la matriz de auto correlación de datos de entrada tienen una amplitud de eigenvalue grande. También, si el ecualizador de retroalimentación de decisiones no ha convergido antes del final de las secuencias de entrenamiento, la forma de dar función objetiva puede cambiar de tal manera que incluye el mínimo local. Estos mínimos locales pueden ser causados por condiciones de canal de ojo cerrado y errores de propagación del ecualizador de retroalimentación de decisiones.

El algoritmo de cuadrado mínimo recursivo (RLS) es bien conocido en evitar estos problemas de convergencia. Sin embargo, el algoritmo del cuadrado mínimo recursivo, en su forma básica, requiere de una inversión de cómputo intensiva de los datos de entrada de la matriz de auto-correlación. Formas basadas en cuadrículas del algoritmo de cuadrado mínimo recursivo evitan la necesidad para esta inversión de matriz. Sin embargo, las formas basadas en cuadrículas del algoritmo de cuadrado mínimo recursivo no se adaptan fácilmente a las ventajas de inicialización de un estimativo de respuesta (CIR) de impulso del canal inicial, aunque esta inicialización pueda ser deseable.

Trabajos recientes en filtrado de rango reducido han conectado el filtro anidado multi estado Wiener (MSNWF) de Golstein y Reed al algoritmo conjugado de gradación. Se ha demostrado que el filtro anidado multi estado Wiener soluciona la ecuación Wiener-

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Hopf en el subespacio Krylov asociado con la matriz de autocorrelación y el director de correlación-cruzada. El el filtro anidado multi estado Wiener entonces se re-formula utilizando la iteración Lanczos. También se ha demostrado que el filtro anidado multi estado Wiener basado en Laczos es equivalente al algoritmo conjugado en gradiente. Como el filtro anidado multi estado Wiener frecuentemente requiere varias dimensiones para acercarse a la ejecución del filtro Winer de rango completo.

Una descripción del algoritmo conjugado de optimización gradiente y muchos de sus propiedades matemáticas se pueden encontrar en "Una introducción a la optimización" por E.K.P. Chong y S. Zak, Nueva York, NY John Wiley & Sons, 1996. El algoritmo descrito se puede aplicar a la optimización de una función de objetivo fijo. Una reseña excelente del filtrado adaptable con el algoritmo conjugado gradiente se encuentra en "análisis de algoritmos conjugados gradientes para el filtrado adaptable" por P. S. Chang and A. N. Willson, Jr., vol. 48, pp. 409-418, IEEE Transacciones en el Procesamiento de Señales, febrero 2000. La riqueza matemática de las relaciones del algoritmo conduce a una variedad de opciones en una implementación del algoritmo conjugado gradiente básico para minimizar una función de objetivo cuadrático. Estas opciones se pueden trasladar a la situación adaptable para suministrar un número de opciones para la implementación del algoritmo. Una característica del algoritmo que también se puede utilizar es que trabaja directamente con las matrices y vectores de correlación. Esta característica hace que la inicialización basada en una respuesta al impulso inicial de canal sea directa, asumiendo que la información para este propósito está disponible.

Como las formas basadas en cuadrículas del algoritmo de cuadrado mínimo recursivo, el algoritmo conjugado gradiente no

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requiere la inversión de la matriz de auto-correlación de datos de entrada. Otra característica del algoritmo conjugado gradiente se puede utilizar reconociendo que la matriz auto-correlacionada de datos de entrada es un producto de dos matrices Toeplitz.

El intento descrito en la Solicitud de Patente de E.U. No. (7227) utiliza estas características para reducir sustancialmente la carga de computación en el algoritmo conjugado gradiente y para incluso eliminar la necesidad de formar la matriz de entrada de auto-correlación. De esta manera, el algoritmo conjugado gradiente mejorado dado a conocer en esa aplicación puede ser utilizado para lograr satisfacer los tiempos de convergencia para los ecualizadores.

La aplicación (7227) también sugiere que el algoritmo conjugado gradiente también se puede utilizar para estimar los canales. El presente invento reduce la complejidad computacional al ejecutar el algoritmo conjugado gradiente con respecto a los estimativos del canal.

Resumen del Invento

De acuerdo con un aspecto del presente invento, un método para procesar una señal recibida y para producir un estimativo de canal que comprende lo siguiente: (a) decodificar la señal recibida y para formar los datos s ; (b) formar una matriz de circunvolución $\hat{S}$ de los datos s ; (c) formar una matriz F de los datos s , donde la matriz F resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución; y, (d) ejecutar un algoritmo conjugado gradiente para determinar el estimativo de canal, donde el algoritmo conjugado gradiente esta basado en la señal recibida y, la matriz $\hat{S}$, y la matriz F .

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De acuerdo con otro aspecto del presente invento, un método para procesar una señal recibida y , que comprende lo siguiente: (a) decodificar la señal recibida y para formar los datos s ; (b) formar una matriz de circunvolución $\hat{S}$ de los datos s ; (c) formar una matriz F de los datos s , donde la matriz F resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución; y, (d) ejecutar un algoritmo conjugado gradiente basado en la señal recibida y , la matriz $\hat{S}$, y la matriz F .

De acuerdo con otro aspecto del presente invento, un método para procesar una señal recibida y , que comprende lo siguiente: (a) decodificar la señal recibida y para formar los datos s ; (b) formar una matriz de circunvolución $\hat{S}$ de los datos s ; (c) formar una matriz F de los datos s , donde la matriz F resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución; y, (d) ejecutar un algoritmo conjugado gradiente basado en la señal recibida y , la matriz $\hat{S}$, y la matriz F , donde el algoritmo conjugado gradiente incluye la formación de FFTs, basados en la señal recibida y , la matriz $\hat{S}$, y la matriz F ; multiplicando los FFTs para formar un producto de multiplicación, y formar un FFT inverso del producto de la multiplicación.

De acuerdo con otro aspecto del presente invento, un método para procesar una señal recibida y para producir un estimativo de canal que comprende lo siguiente: (a) decodificar la señal recibida y para formar los datos s ; (b) formar una matriz de circunvolución $\hat{S}$ de los datos s ; (c) formar una matriz F de los datos s , donde la matriz F resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución; y, (d) ejecutar un algoritmo conjugado gradiente para determinar un estimativo de canal, donde el algoritmo conjugado gradiente esta basado en la señal recibida y , la matriz $\hat{S}$, y la matriz F , donde el algoritmo conjugado gradiente

incluye la formación de FFTs, basados en la señal recibida y, la matriz $\hat{S}$, y la matriz F , multiplicando los FFTs para formar un producto de multiplicación, y formar un FFT inverso del producto de la multiplicación.

Descripción Breve de los Dibujos

Estos y otros rasgos y ventajas se harán aparentes de una consideración detallada del invento cuando se toman en conjunto con los dibujos en donde:

La figura 1 muestra un filtro que se puede utilizar en un ecualizador de retroalimentación de decisión de acuerdo con una implementación del presente intento;

La figura 2 muestra un ecualizador de retroalimentación de decisión de acuerdo con una implementación del presente invento;

La figura 3 muestra una matriz S que se deriva de un vector de decisiones de símbolo de 2496 de largo y que es útil en describir el presente invento;

La figura 4 muestra la descomposición de la matriz S para formar 2 matrices $\hat{S}$ y útiles en la implementación del presente invento; y,

La figura 5 muestra un sistema para determinar un estimativo de canal h basado en las matrices $\hat{S}$ y F .

Descripción Detallada

Tal como se muestra en la figura 1, un filtro 10 se puede utilizar como ecualizador e incluye registros de almacenaje $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$. Por ejemplo, N puede ser 512. Los registros de almacenaje $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$ almacenan los datos recibidos correspondientes $y[n]$, ..., $y[n+N-3]$, $y[n+N-2]$, $y[n+N-1]$. La entrega de cada uno de los

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registros de almacenaje $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$ están acoplados a un multiplicador correspondiente $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$. De acuerdo, los multiplicadores $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$ reciben los datos almacenados de los registros de almacenamiento $12[n]$, ..., $12[n+N-3]$, $12[n+N-2]$, $12[n+N-1]$. Los multiplicadores $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$ también reciben los correspondientes pesos tap $g[n]$, ..., $g[n+N-3]$, $g[n+N-2]$, $g[n+N-1]$. Los multiplicadores $14[n]$, ..., $14[n+N-3]$, $14[n+N-2]$, $14[n+N-1]$ multiplican cada uno de los datos almacenados $y[n]$, ..., $y[n+N-3]$, $y[n+N-2]$, $y[n+N-1]$ por el peso tap correspondiente $g[n]$, ..., $g[n+N-3]$, $g[n+N-2]$, $g[n+N-1]$, y los resultados de la multiplicación los suma la sumatoria 16 para suministrar las salidas del filtro 10. Varios algoritmos como se han descrito anteriormente se han incrementado para fijar los valores de los pesos tap $g[n]$, ..., $g[n+N-3]$, $g[n+N-2]$, $g[n+N-1]$.

Un ecualizador de retroalimentación de decisión 20 se muestra en la figura 2 y incluye un filtro de alimentación 22, un filtro de retroalimentación 24, una sumatoria 26, un dispositivo de decisiones duras 28, y un controlador 30 que computa los pesos tap para el filtro de alimentación veintidós y el filtro de retroalimentación 24. Cada uno de los filtros de alimentación 22 y el filtro de retroalimentación 24 pueden comprender el filtro 10 de la figura 1. El dispositivo de decisión dura 28, por ejemplo, puede ser un decodificador o un cortador.

La salida $\hat{s}[n]$ de la sumatoria 26 es la salida ecualizada del ecualizador de retroalimentación de decisiones 20. El dispositivo de decisión duras 28 (tal como un cortador) determina el valor más cercano $\tilde{s}[n]$ para cada uno de los símbolos suministrados como salida $\hat{s}[n]$ de la sumatoria 26. Estos valores $\tilde{s}[n]$ de Nuevo se reingresan como entradas al filtro de

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retroalimentación 24. El filtro de retroalimentación 24 aplica los pesos tap $\hat{g}_b[n]$ a los valores $\tilde{s}[n]$ y suministrar la salida a la sumatoria 26. de la misma forma, el filtro de alimentación 22 aplica los pesos tap $\hat{g}_f[n]$ a los valores $y[n]$ de la enseñanza recibida y suministra la salida a la sumatoria 26. la sumatoria 26 suma la salida del filtro de retroalimentación 24 con la salida del filtro de alimentación 22 para poder producir un resultado $\tilde{s}[n]$.

La salida $\tilde{s}[n]$ del ecualizador de retroalimentación de decisión 20 está dada por la siguiente ecuación:

$$\begin{aligned}\tilde{s}[n] &= \text{Re}\{g_f^h y[n]\} + g_b^T \tilde{s}[n] \\ &= \mathbf{g}_{FR}^T \mathbf{y}_R[n] + \mathbf{g}_{FI}^T \mathbf{y}_I[n] + \mathbf{g}_b^T \tilde{\mathbf{s}}[n] \quad (1)\end{aligned}$$

donde $\tilde{\mathbf{s}}[n]$ tal como se indica anteriormente estaba definido como los estimativos de símbolo suministrados por el dispositivo de decisión dura 28; donde $\mathbf{g}_{FR} = \text{Re}\{\mathbf{g}_f\}$ es la parte real de los pesos tap $\mathbf{g}_f$ de el filtro de alimentación 22, donde $\mathbf{g}_{FI} = \text{Im}\{\mathbf{g}_f\}$ es la parte imaginaria de los pesos tap $\mathbf{g}_f$ del filtro de alimentación 22, y donde $\mathbf{g}_b$ son los pesos tap del filtro de retroalimentación 24.

Las definiciones dadas por la siguiente ecuación se pueden asumir:

$$\tilde{\mathbf{y}}_f[n] = [\mathbf{y}_R^T[n] \mathbf{y}_I^T[n]]^T \quad (2)$$

$$\tilde{\mathbf{y}}[n] = [\tilde{\mathbf{y}}_f^T[n] \tilde{\mathbf{s}}^T[n]]^T \quad (3)$$

$$\tilde{\mathbf{g}}_f[n] = [\mathbf{g}_{FR}^T[n] \mathbf{g}_{FI}^T[n]]^T \quad (4)$$

$$\tilde{\mathbf{g}}[n] = [\tilde{\mathbf{g}}_f^T[n] \tilde{\mathbf{g}}_b^T[n]]^T \quad (5)$$

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donde $y_r = \text{Re}\{y_f\}$ es la parte real de los símbolos recibidos y_f , y donde $y_i = \text{Im}\{y_f\}$ es la parte imaginaria de los símbolos recibidos y_f . Luego, los símbolos ecualizados $\hat{a}[n]$ suministrados por el ecualizador de retroalimentación de decisión 20 están dados por la siguiente ecuación:

$$\hat{a}[n] = \tilde{g}_r^T \tilde{y}_f[n] + g_s^T \tilde{s}[n] = \tilde{g}^T \tilde{y}[n] \quad (6)$$

La entrada al ecualizador $y[n]$ y los pesos tap del alimentador g_r son valores complejos, los pesos tap de retroalimentación g_s son reales, y la salida $\hat{a}[n]$ del ecualizador de retroalimentación de decisión 20 es real. La parte imaginaria de la salida del filtro de alimentación 22, g_{ri} , se puede ignorar y, por lo tanto, no computarse.

Es deseable determinar el vector de peso tap $\tilde{g}[n]$ de tal manera que el error en la salida del ecualizador de retroalimentación de decisión 20 se minimice de acuerdo con la siguiente ecuación:

$$\tilde{g}[n] = \arg \min_g \frac{1}{2} E\{s[n] - \hat{s}[n]\}^2 \quad (7)$$

$$F(g) = E\{s[n] - \hat{s}[n]\}^2 = \sigma_s^2 - \tilde{g}^H[n] x_{sy} - x_{sy}^H \tilde{g}[n] + \tilde{g}^H[n] R_{yy} g[n] \quad (8)$$

donde σ_s^2 es la variación en el símbolo y es igual a la energía promedio de símbolo R_{yy} es una matriz de auto correlación, y x_{sy} es un vector de correlación cruzada. La matriz de auto correlación R_{yy} y el vector de correlación cruzada x_{sy} se definen. La posición de error $F(g)$ se minimice fijando su gradiente en cero de acuerdo con la siguiente ecuación:

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$$\nabla F(g) = R_{yy}\tilde{g}[n] - r_{sy} = 0 \quad (9)$$

Esta minimización lleva a la conocida ecuación Weiner-Hopf:

$$R_{yy}\tilde{g}[n] = r_{sy} \quad (10)$$

resolviendo para los pesos tap $\tilde{g}[n]$ lleva a la siguiente ecuación:

$$\tilde{g}[n] = R_{yy}^{-1}r_{sy} \quad (11)$$

de esta forma, para calcular el vector de peso tap, el gran sistemas de ecuaciones representado por la ecuación (11) se debe resolver. Esta solución requiere de la inversión de la gran matriz R_{yy} y, por lo tanto, una cantidad considerable de cálculos.

Un método para reducir la cantidad de cálculos involucrados en la solución de sistemas grandes ecuaciones es el algoritmo conjugado gradiente mencionado anteriormente. para establecer la ecuación (11), se requiere de una estimación de la matriz de auto-correlación $R_{yy}[n=0]$ y del vector de correlación cruzada $r_{sy}[n=0]$. Por definición, la matriz de auto-correlación $R_{yy}[n]$ está dada por la siguiente ecuación:

$$R_{yy}[n] = E[\tilde{y}[n]\tilde{y}^H[n]] \quad (12)$$

y el vector de correlación cruzada $r_{sy}[n]$ está dado por la siguiente ecuación:

$$r_{sy}[n] = E[s[n]\tilde{y}[n]] \quad (13)$$

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donde $s[n]$ representa los símbolos transmitidos, E es el operador de expectativa, y H denota una transposición Hermitian.

La matriz de auto-correlación $R_{\tilde{y}\tilde{y}}[n]$ y el vector de correlación cruzada $r_{s\tilde{y}}[n]$ se pueden estimar utilizando el método de factor de olvido de acuerdo con la siguiente ecuación:

$$\begin{aligned}\hat{R}_{\tilde{y}\tilde{y}}[n] &= \sum_{m=0}^{n-1} \gamma^m \tilde{y}[n-m] \tilde{y}^H[n-m] \\ &= \gamma \hat{R}_{\tilde{y}\tilde{y}}[n-1] + \tilde{y}[n] \tilde{y}^H[n] \quad (14)\end{aligned}$$

$$\begin{aligned}\hat{r}_{s\tilde{y}}[n] &= \sum_{m=0}^{n-1} \gamma^m s[n-m] \tilde{y}^H[n-m] \\ &= \gamma \hat{r}_{s\tilde{y}}[n-1] + s[n] \tilde{y}^H[n] \quad (15)\end{aligned}$$

donde $0 < \gamma < 1$ es el factor de olvido. Si γ se fija en 0, las ecuaciones (14) y (15) dan como resultado los estimados de la matriz de auto-correlación $\hat{R}_{\tilde{y}\tilde{y}}$ y el vector de correlación cruzada $\hat{r}_{s\tilde{y}}$ del ejemplo de vector más reciente y .

Determinando los pesos tap g_r para el filtro de alimentación 22 y g_b para y filtro de retroalimentación 24, el controlador 30 inicializa la matriz de auto-correlación $R_{\tilde{y}\tilde{y}}[0]$ utilizando la ecuación (14) y el vector de correlación cruzada $r_{s\tilde{y}}[0]$ utilizando la ecuación (15) ejecutando un promedio sobre un período de tiempo tal como $\frac{1}{4}$ campo de datos. El controlador 30 también inicializa los pesos tap $\tilde{g}_0[0] = 0$ e inicializa el vector de decisión $\mathbf{a}[0]$ al primer N_{fb} de los símbolos de entrenamiento conocidos que están contenidos en la porción del campo sync dedicada campo de datos como se discutió anteriormente.

Aun más, el controlador 30 inicializa el vector de datos $\tilde{y}[0]$ como sigue. El vector de datos $\tilde{y}_f[0]$ como se da en la ecuación (2) se inicializa fijando la parte real $y_R[n=0] = \{y_R[0], \dots, y_R[N_{ff}-1]\}^T$ de tal manera que

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$y_R[0], \dots, y_R[N_{ts} - N_{fb} - 1]$ son la parte real de los datos de entrenamiento restantes (luego de esos en $\tilde{s}[0]$) y $y_R[N_{ts} - N_{fb}], \dots, y_R[N_{ff} - 1]$ son la parte real de los datos recibidos luego de la secuencia de entrenamiento. Esta configuración se repite para $\tilde{y}_i[0]$. Luego, $\tilde{y}_f[0] = [\mathbf{y}_R^T[0] \mathbf{y}_i^T[0]]^T$ se construye como una ecuación dada (2) y $\tilde{y}[0] = [\tilde{y}_f^T[0] \tilde{s}^T[0]]^T$ se construye como una ecuación dada (3) basada en los vectores iniciales $\tilde{y}_f[0]$, $\tilde{y}_R[0]$, y $\tilde{y}_i[0]$ como se dan arriba. En un ambiente de televisión digital ATSC por ejemplo, el número de símbolos de entrenamiento N_{ts} pueden ser 728, el número de taps de alimentación N_{ff} puede ser 512, y el número de taps de retroalimentación N_{fb} puede también ser 512.

Aún más, el controlador 30 inicializan las salidas $\hat{s}[n]$ del ecualizador de retroalimentación de decisión 20 como $\hat{s}[0] = \tilde{\mathbf{g}}_0^T[0] \tilde{\mathbf{y}}[0]$ de acuerdo con adecuación (6), el controlador 30 inicializa una gradiente $t_n[n]$ como $t_0[0] = \hat{R}_{yy}[0] \tilde{\mathbf{g}}_0[0] - \hat{\mathbf{z}}_{xy}[0]$, y el controlador 30 fija la variable n en 0.

El controlador 30 luego ejecuta un algoritmo conjugado gradiente una vez por cada elemento de datos recibido, donde cada iteración del algoritmo se inicia tan pronto ocurre el cambio de un elemento de datos nuevo que se recibe en el filtro de alimentación 22. Los parámetros del algoritmo conjugado gradiente como se describen más abajo están definidos en la tabla.

Tabla

Nombre del parámetro	Notación del ecualizador
Argumento de la función del objeto (ecualizador)	$\mathbf{g}_k[n]$
Tamaño del paso del argumento	$\alpha_k[n]$
Gradiente	$\mathbf{t}_k[n]$
Direcciones conjugadas	$\mathbf{b}_k[n]$

Tamaño del paso de las direcciones conjugadas	$\beta_k[n]$
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este algoritmo de gradiente conjugado es como sigue:

1. dirección conjugada calculada $b_k[n]$

si $n \bmod (N_{\text{taps}}) = 0$

$$b_0[n] = -t_0[0] \quad (\text{reanudar})$$

o

$$\beta_0[n] = \frac{(t_0[n] - t_{-1}[n])^H t_0[n]}{(t_0[n] - t_{-1}[n])^H b_{-1}[n]}$$

$$b_0[n] = -t_0[n] + \beta_0[n] b_{-1}[n]$$

2. tamaño del paso del argumento calculado $\alpha_k[n]$

$$\alpha_0[n] = \frac{-b_0^H[n] t_0[n]}{b_0^H[n] \hat{R}_{yy}[n] b_0[n]}$$

3. actualización del tap

$$\tilde{g}_1[n] = \tilde{g}_0[n] + \alpha_0[n] b_0[n]$$

4. $n = n + 1$

$$\tilde{g}_0[n] = \tilde{g}_1[n - 1]$$

$$t_{-1}[n] = t_0[n - 1]$$

$$b_{-1}[n] = b_0[n - 1]$$

5. Actualización del vector de entrada $\tilde{y}[n]$ con el siguiente elemento de datos recibidos y la actualización $\hat{R}_{yy}[n]$ por ecuación

(14)

6. calcula la salida del ecualizador de retroalimentación de decisión 20

$$\hat{s}[n] = \tilde{g}_0^T[n] \tilde{y}[n]$$

7. calcula la gradiente $t_k[n]$

$$t_0[n] = t_{-1}[n] + \alpha_0[n] \hat{R}_{yy}[n] b_0[n]$$

8. Regresa al paso 1

La parte más grande de los cálculos requeridos por este algoritmo conjugado gradiente es una contribución de la multiplicación de la matriz de vectores $\hat{R}_{yy}[n]b_0[n]$ de los pasos 2 y 7. Estas multiplicaciones requieren la actualización de la matriz $\hat{R}_{yy}[n]$ desde el vector $\tilde{y}[n]$ como en adecuación (14). esta actualización requiere una multiplicación por cada uno de los elementos de la matriz N_{taps}^2 , un total de N_{taps}^2 multiplicaciones. Luego, la multiplicación de la matriz de vectores requiere N_{taps}^2 multiplicaciones. De esta forma, el cálculo total $N_{\text{taps}}^2 + N_{\text{taps}}^2 = 2N_{\text{taps}}^2$ multiplicaciones.

Esta carga en el cálculo se puede disminuir cómo se explica a continuación.

Una multiplicación de la matriz-vector $a = Yb$, donde Y es una matriz de circunvolución, b es un vector, y a es el resultado de la multiplicación, equivalente a $a[n] = y[n] * b[n]$ donde $*$ denota una circunvolución lineal. Estas ecuaciones se pueden representar en la forma de una matriz por la siguiente ecuación:

$$\begin{bmatrix} a[0] \\ a[1] \\ a[2] \end{bmatrix} = \begin{bmatrix} y[0] & y[-1] & y[-2] \\ y[1] & y[0] & y[-1] \\ y[2] & y[1] & y[0] \end{bmatrix} \begin{bmatrix} b[0] \\ b[1] \\ b[2] \end{bmatrix} \quad (16)$$

donde n se escoge como un número pequeño por razones de explicación. Las columnas y las filas de la matriz Y en la ecuación (16) se pueden extender en forma circular de acuerdo con la siguiente ecuación:

$$\begin{bmatrix} a[0] \\ a[1] \\ a[2] \\ dc \\ dc \end{bmatrix} = \begin{bmatrix} y[0] & y[-1] & y[-2] & y[2] & y[1] \\ y[1] & y[0] & y[-1] & y[-2] & y[2] \\ y[2] & y[1] & y[0] & y[-1] & y[-2] \\ y[-2] & y[2] & y[1] & y[0] & y[-1] \\ y[-1] & y[-2] & y[2] & y[1] & y[0] \end{bmatrix} \begin{bmatrix} b[0] \\ b[1] \\ b[2] \\ 0 \\ 0 \end{bmatrix} \quad (17)$$

donde dc significa no me importa. La ecuación (17) es equivalente a $a[n] = y[n] \otimes b[n]$ donde $\otimes$ denota una circunvolución circular. La circunvolución circular es equivalente a la siguiente operación:

$$\mathcal{Y} = \text{FFT}(\mathbf{y}) \quad (18)$$

$$\mathcal{B} = \text{FFT}(\mathbf{b}) \quad (19)$$

$$\mathcal{A} = \mathcal{Y} \times \mathcal{B} \quad (20)$$

$$\mathbf{a} = \text{IFFT}(\mathcal{A}) \quad (21)$$

donde FFT en ecuación (18) y (19) indica una operación Fast Fourier Transform, IFFT en ecuación (21) indica una operación inverse de Fast Fourier Transform, y $\times$ en la ecuación (20) indica una multiplicación de vector tal en los elementos del vector resultante están dados por $\mathcal{Y}_0 \times \mathcal{B}_0$, $\mathcal{Y}_1 \times \mathcal{B}_1$, $\mathcal{Y}_2 \times \mathcal{B}_2$, etc.

Un embellecimiento del proceso anterior involucra un relleno de 0 para las columnas con un largo 2^n . Este relleno con 0 como es bien conocido, permite una eficiencia de computo FFT. De acuerdo, la multiplicación de la matriz-vector $\mathbf{a} = \mathbf{Yb}$ se puede escribir con relleno de 0 en la forma de una matriz de acuerdo con la siguiente ecuación:

$$\begin{bmatrix} a[0] \\ a[1] \\ a[2] \\ dc \\ dc \\ dc \\ dc \\ dc \end{bmatrix} = \begin{bmatrix} y[0] & y[-1] & y[-2] & 0 & 0 & 0 & y[2] & y[1] \\ y[1] & y[0] & y[-1] & y[-2] & 0 & 0 & 0 & y[2] \\ y[2] & y[1] & y[0] & y[-1] & y[-2] & 0 & 0 & 0 \\ 0 & y[2] & y[1] & y[0] & y[-1] & y[-2] & 0 & 0 \\ 0 & 0 & y[2] & y[1] & y[0] & y[-1] & y[-2] & 0 \\ 0 & 0 & 0 & y[2] & y[1] & y[0] & y[-1] & y[-2] \\ y[-2] & 0 & 0 & 0 & y[2] & y[1] & y[0] & y[-1] \\ y[-1] & y[-2] & 0 & 0 & 0 & y[2] & y[1] & y[0] \end{bmatrix} \begin{bmatrix} b[0] \\ b[1] \\ b[2] \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (22)$$

Se observa que la matriz $\mathbf{Y}$ tiene una estructura Toeplitz, y que $\mathbf{Y}^H$ es también una matriz Toeplitz. (H denota una transposición Hermitian.)

Se demostrará que la necesidad para ambas, la actualización $\mathbf{R}_{yy}$ y la multiplicación $\mathbf{R}_{yy}\mathbf{b}$ se puede eliminar, y que la matriz $\mathbf{R}_{yy}$ no se necesita siquiera crear.

Deje $\gamma = 0$ para la ecuación (14). Es bien sabido que

$$\mathbf{R}_{yy} = \tilde{\mathbf{Y}}\tilde{\mathbf{Y}}^H = \mathbf{Y}\mathbf{Y}^H$$

donde $\mathbf{Y}$ es una matriz de circunvolución basada en el vector $\mathbf{y}$ (prestadores, $\mathbf{y}$ forma la primera columna de la matriz $\mathbf{Y}$). El vector $\tilde{\mathbf{y}}$ se puede rellenar con 0 hasta lo largo de 2^n . Deje $\tilde{\mathbf{y}}'$ ser la primeras columnas de $\mathbf{Y}^H$. para la operación de gradiente conjugado, los cálculos dados por la siguiente ecuación se deben hacer:

$$\mathbf{v} = \hat{\mathbf{R}}_{yy}[n]\mathbf{b}_0[n] = \mathbf{Y}^H\mathbf{Y}\mathbf{b}_0[n] \quad (23)$$

Para calcular $\mathbf{v}$, la cantidad $\mathbf{Y}^H\mathbf{b}_0[n]$ se debe primero calcular de acuerdo a la siguiente ecuación:

$$\mathbf{Y} = \text{FFT}(\mathbf{y}) \quad (24)$$

$$\mathbf{b} = \text{FFT}(\mathbf{b}) \quad (25)$$

$$X = Y \times \mathcal{O} \quad (26)$$

$$x = \text{IDFT}(X) \quad (27)$$

$$x_N = \text{primeros } N \text{ elementos de } x, \text{ elementos restantes} = 0 \quad (28)$$

$$X_N = \text{FFT}(x_N) \quad (29)$$

donde x en adecuación (26) denota una multiplicación de vector al punto. Luego, $Y \times X_N$ se calcula de acuerdo con la siguiente ecuación:

$$Y' = \text{FFT}(Y') \quad (30)$$

$$V = Y' \times X \quad (31)$$

$$v = \text{IDFT}(V) \quad (32)$$

$$v_N = \text{primeros } N \text{ elementos de } v, \text{ elementos restantes} = 0 \quad (33)$$

El número de multiplicaciones en este proceso, particularmente para grandes N_{taps} , es mucho menos de $2N_{\text{taps}}^2$.

En el proceso dado por las ecuaciones (24)-(33), b es el vector b_0 del algoritmo de ocho pasos descrito anteriormente, y es un vector de símbolos recibidos como se describen anteriormente, y v_N es la cantidad $\hat{R}_{yy}[n]b_0[n]$ en los pasos 2 y 7 del algoritmo conjugado gradiente descrito anteriormente.

El algoritmo conjugado gradiente descrito arriba se puede utilizar para hacer otros cálculos. Por ejemplo, el algoritmo conjugado de gradiente escrito arriba se puede utilizarse para calcular estimativos de actualización de canal de impulsos de respuestas. Este cálculo puede ser útil, por ejemplo, porque no todos conocidos alternos para calcular los pesos tap del ecualizador MMSE (Minimum Mean Square Error) están basados en tener un estimativo de respuesta del impulso del canal.

El método conjugado gradiente descrito anteriormente permite el cálculo continuo de actualizaciones de estimativos de respuestas de impulso de canal entre sesiones de entrenamiento (únicamente cuando se reciben símbolos desconocidos a priori) en un receptor para un canal multi-camino que varía. Este cálculo utiliza un estimativo de respuesta de impulse de canal cuadrado calculado de un sistema de ecuaciones determinado. Cambios dinámicos a la respuesta del impulso de canal se pueden seguir utilizando un decodificador trellis de receptor en símbolos de entrada para formar una secuencia larga de símbolos perfectamente conocidos. Esta secuencia debe tener relativamente pocos errores.

La respuesta de impulso de canal que se va a estimar es de un largo $L_h = L_{ha} + L_{hc} + 1$ donde L_{ha} es el largo de la parte anti-causal de la respuesta del impulse de canal y L_{hc} es el largo de la parte causal deben dar respuesta de impulse de canal.

Las decisiones confiables del decodificador trellis en los símbolos de entrada es un vector que se designa acá como $\mathbf{s}$ de largo L_s . Una matriz Toeplitz S se puede definir basada en este vector $\mathbf{s}$ de acuerdo con la siguiente ecuación:

$$\mathbf{S} = \begin{bmatrix} S_{L_h-1} & S_{L_h-2} & - & - & - & - & - & S_0 \\ - & S_{L_h-1} & - & - & - & - & - & - \\ - & - & & & & & - & - \\ - & - & & & & & - & - \\ - & - & & & & & - & S_{L_h-1} \\ - & - & & & & & - & - \\ - & - & & & & S_{L_s-L_h} & - & - \\ S_{L_s-1} & S_{L_s-2} & - & - & - & - & - & S_{L_s-L_h} \end{bmatrix} \quad (34)$$

donde los elementos de la matriz de la ecuación(34) son reales y consisten de las decisiones simbólicas del vector $\mathbf{s}$.

Para asegurar un sistema sobre-determinado de ecuaciones, la siguiente desigualdad es necesaria:

$$L_s \geq 2L_h - 1. \quad (35)$$

La matriz S es de una dimensión $(L_s - L_h + 1) \times L_h$ con $(L_s - L_h + 1) \geq L_h$. El vector de la señal recibida y con elementos de datos y_i para $L_{hc} \leq i \leq (L_s - L_{ha} - 1)$ donde y_i es el símbolo recibido correspondiente a la decisión de símbolo de entrada s_i . Luego, $y = Sh + v$ donde h es el vector de respuesta del impulse del canal largo L_h y v es un vector de ruido blanco.

Las soluciones cuadradas mínimas para h están dadas por la siguiente ecuación:

$$\hat{h}_{LS} = (S^T S)^{-1} S^T y. \quad (36)$$

Utilizando un decodificador confiable trellis De decisiones de símbolos de entrada, debe haber suficiente soporte para calcular un estimativo de respuesta de impulse de canal del largo requerido. Tal como lo requiere la desigualdad (45), el vector s de la decisión de símbolo debe ser por lo menos el doble de largo que la respuesta de impulse de canal que se estima.

Para una aplicación de recepción 8 VSB se pueden asumir los siguientes parámetros: $L_h = 512$, $L_{ha} = 63$, $L_{hc} = 448$, y $L_s = 2496$. El vector s se puede formar de una secuencia de decisiones de decodificador trellis en los símbolos de entrada. Normalmente, el decodificador únicamente tomará decisiones de salida en pares de bit, pero también puede tomar decisiones igualmente confiables en los símbolos de entrada.

El vector $\mathbf{s}$, por ejemplo, se puede seleccionar cómo tres segmentos con un largo de ($L_s = 2496$ simbolos). Por lo tanto, tres segmentos de datos se pueden utilizar para producir una actualización única de estimativos de respuesta de impulse de canal. Una actualización de estimativo de respuesta de impulso de canal nueva se puede obtener una vez por segmento procediendo a la manera de una ventana que corre. Opcionalmente, varias actualizaciones de estimativos de respuestas de impulso de canal se pueden promediar para mejorar la precisión de los estimativos de respuesta de impulso de canal si son necesarios. Claro está, que este paso adicional de promediar puede ser un problema si la respuesta de impulso de canal cambian rápidamente.

Un vector $\mathbf{b}$ con menos de tres segmentos de decisión de símbolo se puede utilizar, pero como se expresó en la desigualdad (35), el largo del vector $\mathbf{b}$ debe ser por lo menos el doble del largo que dar respuesta de impulse de canal que se va a estimar. Los lectores más largos $\mathbf{b}$ ayuda en a disminuir el efecto adverso del ruido aditivo blanco Gaussian en el canal.

El sistema de ecuaciones representado por la ecuación (36) se puede resolver utilizando el alcoholismo conjugado gradiente en una forma similar a la descrita previamente para resolver la ecuación (11) para el peso tap MMSE donde $\mathbf{S}^T \mathbf{y}$ en la ecuación (36) tomar el lugar de $\mathbf{r}_s \mathbf{y}$ en la ecuación (11), donde $\mathbf{S}^T \mathbf{s}$ en la ecuación (36) , en lugar de R_{yy} en la ecuación (11), y donde $\hat{\mathbf{h}}_{LS}$ en la ecuación (36) , en lugar de $\tilde{\mathbf{g}}$ en la ecuación (11).

Entonces, el algoritmo conjugado gradiente escrito más arriba puede facilitar el cómputo de $(\mathbf{S}^T \mathbf{s}) \mathbf{b}$ para el caso de un estimativo de respuesta de impulse de canal así como facilitó el cómputo de $R_{yy} \mathbf{b}$ para el cálculo de pesos tap.

Reduciendo adicionalmente la complejidad del cómputo necesario para implementar las multiplicaciones matriz-vector requeridas para ejecutar el algoritmo conjugado gradiente, tal como cuando el algoritmo conjugado gradiente se utiliza para estimar un canal como se describió arriba, es útil notar que el algoritmo conjugado gradiente se puede utilizar para resolver el siguiente sistema línea al para el estimativo de canal h :

$$Sh = y. \quad (37)$$

La matriz S es una $m \times n$ (e.g., 1985×512) matriz Toeplitz derivada de las decisiones de símbolo, y y es un vector (el vector de observación) derivado de los datos recibidos. Las decisiones de símbolo se pueden hacer, por ejemplo, con un decodificador trellis o un cortador.

El sistema se puede escribir como $S^T Sh = S^T y$ multiplicando ambos lados de la ecuación (37) por S^T . El algoritmo conjugado gradiente se puede utilizar para resolver este sistema para el estimativo del canal h de acuerdo con los siguientes pasos:

- (1) Calcular el vector residual r_1 que representa al sistema $S^T y - S^T Sh_1$ de la siguiente manera:

$$r_1 = S^T y - S^T Sh_1 \quad (38)$$

donde r_1 es un vector 512×1 y donde h_1 es el estimativo del canal previo que se puede inicializar a cualquier valor deseado tal como $h_1 = 0$.

- (2) Para $k = 1$ a n , en forma iterativa calcular lo siguiente:

$$(a) \mathbf{d}_k = \mathbf{r}_k + \beta_k \mathbf{d}_{k-1} \quad (39)$$

donde $\beta_1 = 0$, $\beta_{k \geq 2} = \frac{\mathbf{r}_k^T \cdot \mathbf{r}_k}{\mathbf{r}_{k-1}^T \cdot \mathbf{r}_{k-1}}$, y donde $\mathbf{d}_k$ es un vector 512×1 ;

$$(b) \mathbf{h}_{k+1} = \mathbf{h}_k + \alpha_k \mathbf{d}_k \quad (40)$$

donde $\mathbf{h}_{k+1}$ es un vector 512×1 , donde α_k es 1×1 , donde

$\alpha_k = \frac{\mathbf{r}_k^T \cdot \mathbf{r}_k}{\mathbf{d}_k \cdot \mathbf{q}_k}$, donde $\mathbf{q}_k = \mathbf{S}^T \mathbf{S} \mathbf{d}_k$, donde $\mathbf{S}^T$ es una matriz 512×1985 , y

donde $\mathbf{S}$ es una matriz 1985×512 ; y,

$$(c) \mathbf{r}_{k+1} = \mathbf{r}_k - \alpha_k \mathbf{q}_k \quad (41)$$

donde $\mathbf{r}_{k+1}$ es un vector 512×1 .

Los pasos 2(a) - (c) se repiten suficientes intereses hasta que la decisión deseada se logra para el estimativo de canal $\mathbf{h}$. El índice k , por ejemplo, se puede incrementar en un valor hasta 5 aunque cualquier otro valor deseado se puede utilizar. Se verá que el paso 2(b) incluye la multiplicación del vector matriz $\mathbf{A}^T \mathbf{A} \mathbf{d}_k$. Esta operación es un cómputo móvil complejo. Una técnica para reducir la complejidad de cómputo de implementar esta operación se sugiere más arriba con respecto a las ecuaciones (23) - (33). El presente invento mejora esta técnica.

Un enfoque directo de esta técnica sería formar un FFT de cada uno de los tres componentes $\mathbf{S}^T$, $\mathbf{S}$, y $\mathbf{d}_k$, el vector de punto inteligente multiplica los tres FFTs resultantes, y luego toma el FFT inverso del resultado de la multiplicación. Esta serie de cálculos, sin embargo no suministrando una respuesta precisa porque la matriz $\mathbf{S}$ no es una verdadera matriz de circunvolución, y el IFFT del producto triple, por lo tanto, produce un resultado corrupto. La figura 3 muestra la matriz $\mathbf{S}$ que se deriva de un vector de

símbolo de decisión 2496 (tres segmentos de un campo VSB). La 1ª fila de la matriz S contienen la decisión del símbolo 512 en la primera columna hasta la decisión de símbolo 1 en la última columna, la 2ª fila de la matriz S contienen la decisión de símbolo 513 en la primera columna hasta la decisión de símbolo dos en la última columna, y así. la porción de la matriz S que previenen que sería una verdadera matriz de circunvolución son las áreas triangulares 40 y 42. Las líneas diagonales en la figura 3 son paralelas.

De acuerdo con el presente invento, y como se muestra en la figura 4 la matriz S de la figura 3 se descompone para formar dos matrices $\hat{S}$ y F de tal manera que la matriz $\hat{S}$ es una verdadera matriz de circunvolución. de esta manera, el área 44 de la matriz $\hat{S}$ es la misma que el área 44 de la matriz S , y las decisiones de símbolo en las áreas 40 y 42 de la matriz $\hat{S}$ se fijan en 0. La decisión de símbolo en el área 44 de la matriz F se fijan en cero, y las áreas 40 y 42 de la matriz F son las mismas que las áreas 40 y 42 de la matriz S . Un Nuevo sistema línea al basado en $\hat{S}$ y F se definen luego para resolver el estimativo de canal h .

Ventajosamente, las operaciones FFT de $\hat{S}$ se pueden utilizar para implementar la multiplicación de la matriz de vector del algoritmo conjugado gradiente sin corrupción porque $\hat{S}$ es verdaderamente una matriz de circunvolución.

Específicamente, porque el sistema lineal está definido por $y = Sh$, resulta la siguiente ecuación:

$$y = (\hat{S} + F)h \quad (42)$$

$$y = \hat{S}h + Fh \quad (43)$$

$$y - Fh = \hat{S}h \quad (44)$$

Definiendo $\hat{\mathbf{y}} = \mathbf{y} - \mathbf{F}\mathbf{h}_1$, donde $\mathbf{h}_1$ es un estimativo de canal previo derivado por medio de cualquier técnica conocida, la nueva expresión del sistema se convierte en $\hat{\mathbf{y}} = \hat{\mathbf{S}}\mathbf{h}$. Esta expresión se puede escribir como $\hat{\mathbf{S}}^T \hat{\mathbf{y}} = \hat{\mathbf{S}}^T \hat{\mathbf{S}}\mathbf{h}$. El algoritmo conjugado gradiente se puede entonces aplicar para resolver $\mathbf{h}$ en el nuevo sistema de la siguiente manera:

- (1) $\hat{\mathbf{y}} = \mathbf{y} - \mathbf{F}\mathbf{h}_1$,
 $\mathbf{r}_1 = \hat{\mathbf{S}}^T \hat{\mathbf{y}} - \hat{\mathbf{S}}^T \hat{\mathbf{S}}\mathbf{h}_1$
- (2) Para $k = 1$ a n , calcular en forma iterativa
 - (a) $\mathbf{d}_k = \mathbf{r}_k + \beta_k \mathbf{d}_{k-1}$
 - (b) $\mathbf{h}_{k+1} = \mathbf{h}_k + \alpha_k \mathbf{d}_k$
 - (c) $\mathbf{r}_{k+1} = \mathbf{r}_k - \alpha_k \mathbf{q}_k$

donde $\mathbf{h}_1$ se puede inicializar cualquier valor deseado como 0, y donde $\mathbf{d}_0$, $\mathbf{r}_0$, β_k , y α_k son como se definieron anteriormente.

Como se indica arriba la siguiente ecuación (40), paso 2(b) incluye el cálculo de la expresión $\mathbf{q}_k = \hat{\mathbf{S}}^T \hat{\mathbf{S}}\mathbf{d}_k$. Esta multiplicación se puede resolver en forma conveniente formando el FFT de (i) la matriz $\hat{\mathbf{S}}$, (ii) $\mathbf{d}_k$, y (iii) $\hat{\mathbf{S}}^T$. Los tres resultados FFT son vectores de punto inteligente multiplicados, y la FFT inversa (IFFT) se toma del resultado de la multiplicación. Los primeros N elementos de este IFFT producen $\mathbf{q}_k$. N , por ejemplo, puede tener un valor de 512 para una FFT/IFFT de 1024 puntos. Sin embargo, N puede tener otros valores como se desee.

La figura 5 muestra un sistema 50 para implementar esta técnica de tal manera que determine un estimativo de canal $\mathbf{h}$. el sistema 50 incluye un decodificador 52 que decodifica los datos recibidos y para producir los símbolos s . Como se describió arriba, el decodificador 52 puede ser un decodificador trellis. Sin

embargo, el decodificador 52 puede ser otras formas de decodificador tales como cortadores. Un estimador de canal 54 produce un estimativo de canal h utilizando la técnica descrita arriba.

Modificaciones del presente invento le ocurrirán a aquellos practicando en el arte del presente invento. Por ejemplo, el Fast Fourier Transform (FFT) y el Fast Fourier Transform inverso (IFFT) se utilizan en el presente invento como se describió más arriba. En forma alterna el Discrete Fourier Transform (DFT) y el Discrete Fourier Transform (IDFT) inverso se pueden utilizar interés del FFT y el IFFT.

De esta manera, la descripción del presente invento se debe interpretar como ilustrativa únicamente y tiene el propósito de enseñar aquellos diestros en el arte la mejor manera de llevar a cabo el invento. Los detalles se pueden variar sustancialmente sin alejarse del espíritu del invento, y se reserva el uso exclusivo de todas las modificaciones que están dentro del alcance de las reivindicaciones anexas.

Nosotros reivindicamos:

1. Un método para procesar una señal que se recibe para producir un estimativo de canal que comprende:

(a) decodificar la señal recibida y para formar los datos s ;

(b) formar una matriz de circunvolución $\hat{S}$ de los datos s ;

(c) formar una matriz F de los datos s , donde la matriz F resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución; y,

(d) ejecutar un algoritmo conjugado gradiente para determinar este estimativo de canal, donde el algoritmo conjugado gradiente está basado en la señal que se recibe y, la matriz $\hat{S}$, y la matriz F .

2. el método de la reivindicación 1 donde la ejecución de un algoritmo conjugado gradiente comprende determinar una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y, la matriz $\hat{S}$, y la matriz F , y donde q_k se determina formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando en primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar una FFT inverso del resultado de la multiplicación.

3. El método de la reivindicación 1 donde la formación de la matriz $\hat{S}$ de los datos s comprende:

formar una matriz S de los datos s ; y,

formar una matriz $\hat{S}$ de la matriz S fijando ciertos valores de la matriz S en cero;

y donde la formación de la matriz de los datos s comprende:

formar la matriz F de la matriz S fijando en cero los valores de la matriz S que no se fijaron en cero durante la formación de la matriz $\hat{S}$.

4. El método de la reivindicación 3 donde la ejecución de un algoritmo conjugado gradiente comprende determinar una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y , la matriz $\hat{S}$, y la matriz F , y donde q_k se determina formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar una FFT inverso del resultado de la multiplicación.

5. El ecualizador de la reivindicación 1 donde ejecutar un algoritmo conjugado gradiente para determinar el estimativo del canal h comprende ejecutar el siguiente algoritmo:

$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S} h_1$$

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(2) Para $k = 1$ a n , calcular iterativamente

$$(a) \quad d_k = r_k + \beta_k d_{k-1}$$

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad r_{k+1} = r_k - \alpha_k q_{k-1}$$

donde h_1 es un valor inicial del estimativo de canal, donde

$$\beta_1 = 0, \beta_{k \geq 2} = \frac{r_k^T \cdot r_k}{r_{k-1}^T \cdot r_{k-1}}, \text{ donde } \alpha_k = \frac{r_k^T \cdot r_k}{d_k \cdot q_k}, \text{ donde } q_k = S^T S d_k.$$

6. El método de las reivindicaciones 5 donde la ejecución de un algoritmo conjugado gradiente comprende determinar la cantidad q_k formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, si formando un FFT inverso del resultado de la multiplicación.

7. El método de las reivindicaciones 5 donde la formación de la matriz $\hat{S}$ de los datos s comprende:

formar una matriz S de los datos s ; si,

formar una matriz $\hat{S}$ de la matriz S fijando ciertos valores de la matriz S en cero;

y donde la formación de la matriz de los datos s comprende:

formar la matriz F de la matriz S fijando en cero los valores de la matriz S que no se fijaron en 0 durante la formación de la matriz $\hat{S}$.

8. El método de la reivindicación 7 donde la ejecución de un algoritmo conjugado gradiente comprende determinar la cantidad

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q_k formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar un FFT inverso del resultado de la multiplicación.

9. Un método para procesar la señal recibida que comprende:

(a) decodificar una señal recibida y para formar los datos s ;

(b) formar una matriz de circunvolución $\hat{S}$ de los datos s ;

(c) transformar una matriz F de los datos s , donde la matriz F resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución; y,

(d) ejecutar un algoritmo conjugado gradiente basado en la señal recibida y , la matriz $\hat{S}$, y la matriz F .

10. El método de la reivindicación 9 en donde la ejecución de un algoritmo conjugado gradiente comprende determinar una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y , la matriz $\hat{S}$, y la matriz F , y donde q_k se determina formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar un FFT inverso del resultado de la multiplicación.

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11. El método de la reivindicación 9 donde la formación de la matriz $\hat{S}$ de los datos s comprende:

formar una matriz S de los datos s ; y,

formar una matriz $\hat{S}$ de la matriz S fijando ciertos valores de la matriz S en cero;

y donde la formación de la matriz F de los datos s comprende:

formar la matriz F de la matriz S fijando en cero los valores de la matriz S que no se fijaron en cero durante la formación de la matriz $\hat{S}$.

12. El método de la reivindicación 11 en donde la ejecución de un algoritmo conjugado gradiente comprende determinar una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y , la matriz $\hat{S}$, y la matriz F , y donde q_k se determina formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar un FFT inverso del resultado de la multiplicación.

13. Un método para procesar una señal recibida y que comprende:

(a) decodificar la señal recibida y para formar los datos

s ;

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y donde la formación de la matriz F de los datos s
comprende:

formar la matriz F de la matriz S fijando en cero los
valores de la matriz S que no se fijaron en cero durante la
formación de la matriz $\hat{S}$.

16. El método de la reivindicación 15 en donde la
ejecución de un algoritmo conjugado gradiente comprende determinar
una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y, la matriz $\hat{S}$, y
la matriz F, y donde q_k se determina formando un primer FFT de la
matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un
tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs
para producir un resultado de multiplicación, y formar un FFT
inverso del resultado de la multiplicación.

17. Un método para procesar la señal recibida y para producir un
estimativo de canal y que comprende:

(a) decodificar la señal recibida y para formar los datos
s;

(b) formar una matriz de circunvolución $\hat{S}$ de los datos
s;

(c) formar una matriz F de los datos s, donde la matriz F
resulta de formar la matriz $\hat{S}$ como una matriz de circunvolución;
y,

(d) ejecutar un algoritmo conjugado gradiente basado en
la señal recibida y, la matriz $\hat{S}$, y la matriz F, donde el
algoritmo conjugado gradiente incluye la formación de FFTs basados

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en la señal recibida y, la matriz $\hat{S}$, la matriz F , multiplicando los FFTs para formar un producto de multiplicación, y formar un FFT inverso del producto de la multiplicación.

18. El método de la reivindicación 17 en donde la ejecución de un algoritmo conjugado gradiente comprende determinar una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y, la matriz $\hat{S}$, y la matriz F , y donde q_k se determina formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar un FFT inverso del resultado de la multiplicación.

19. El método de la reivindicación 17 donde la formación de la matriz $\hat{S}$ de los datos s comprende:

formar una matriz S de los datos s ; y,

formar una matriz $\hat{S}$ de la matriz S fijando ciertos valores de la matriz S en cero;

y donde la formación de la matriz F de los datos s comprende:

formar la matriz F de la matriz S fijando en cero los valores de la matriz S que no se fijaron en cero durante la formación de la matriz $\hat{S}$.

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20. El método de la reivindicación 19 en donde la ejecución de un algoritmo conjugado gradiente comprende determinar una cantidad q_k de acuerdo con la siguiente ecuación:

$$q_k = \hat{S}^T \hat{S} d_k,$$

donde d_k es dependiente de la señal que se recibe y, la matriz $\hat{S}$, y la matriz F , y donde q_k se determina formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar un FFT inverso del resultado de la multiplicación.

21. El ecualizador de la reivindicación 17 donde la ejecución de un algoritmo conjugado gradiente para determinar un estimativo de canal h comprende la ejecución del siguiente algoritmo:

$$(1) \quad \hat{y} = y - Fh_1,$$

$$r_1 = \hat{S}^T \hat{y} - \hat{S}^T \hat{S} h_1$$

(2) Para $k = 1$ a n , calcular en forma iterativa

$$(a) \quad d_k = r_k + \beta_k d_{k-1}$$

$$(b) \quad h_{k+1} = h_k + \alpha_k d_k$$

$$(c) \quad r_{k+1} = r_k - \alpha_k q_{k-1}$$

donde h_1 es un valor inicial del estimativo de canal, donde

$$\beta_1 = 0, \beta_{k \geq 2} = \frac{r_k^T \cdot r_k}{r_{k-1}^T \cdot r_{k-1}}, \text{ donde } \alpha_k = \frac{r_k^T \cdot r_k}{d_k \cdot q_k}, \text{ donde } q_k = S^T S d_k.$$

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22. El método de la reivindicación 21 donde la ejecución de un algoritmo conjugado gradiente comprende la determinación de la cantidad q_k formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar una FFT inverso del resultado de la multiplicación.

23. El método de la reivindicación 21 donde la formación de la matriz $\hat{S}$ de los datos s comprende:

formar una matriz S de los datos s ; y,

formar una matriz $\hat{S}$ de la matriz S fijando ciertos valores de la matriz S en cero;

y donde la formación de la matriz F de los datos s comprende:

formar la matriz F de la matriz S fijando en cero los valores de la matriz S que no se fijaron en cero durante la formación de la matriz $\hat{S}$.

24. El método de la reivindicación 23 donde la ejecución de un algoritmo conjugado gradiente comprende la determinación de la cantidad q_k formando un primer FFT de la matriz $\hat{S}$, formando un segundo FFT de la matriz $\hat{S}^T$, formando un tercer FFT de d_k , multiplicando el primer, segundo, tercer FFTs para producir un resultado de multiplicación, y formar una FFT inverso del resultado de la multiplicación.

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Resumen

Formando un estimativo de canal, una señal recibida y se decodifica para formar los datos s , de los datos s se forma una matriz de circunvolución $\hat{S}$ se forma de los datos s , una matriz F se forma de los datos s de tal manera que la matriz F resulta de la formación de la matriz como una matriz $\hat{S}$ de circunvolución, y un algoritmo conjugado gradiente se ejecuta para determinar el estimativo de canal. El algoritmo conjugado gradiente esta basado en la señal recibida y, la matriz $\hat{S}$ y la matriz F .

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SUPERINTENDENCIA DE INDUSTRIA Y COMERCIO

NIT : 800176089-2

SUPERINTENDENCIA DE INDUSTRIA Y COMERCIO
Rad: 06-054051- -00000-0000
Fec: 2006-06-05 17:28:36 Dep. 2020 NUECREACIONES
Tra. 11 PATENTENY
Eve: 370 FABENACIONALH
Act 411 PRESENTACION Folios: 200

RECIBO OFICIAL DE CAJA : 06 - 42,489
FECHA : JUNIO 5 DE 2006

***** CONSIGNACION *****

DEPOSITANTE	TIPO PAGO	BANCO	CUENTA	No. PAGO	FECHA PGO	Vr. PAGO
CARDENAS Y CARDENAS P.I. CONSIGNACION		BANCO POPULAR	050-00110-6	36043078	05/06/2006	6,013,000.00

***** CONCEPTO *****

CANT.	RENTISTICO	CONCEPTO	TOTAL CONCEPTO
1	50005-01-01	SOLICITUDES	
		1 TRAMITES DE SOL. DE PATENTE DE IN	463,000.00
		TOTAL :	\$ 463,000.00

CON: CUATROCIENTOS SESENTA Y TRES MIL PESOS

RESPONSABLE : _____

RECIBO DE CAJA APLICADO AL EXPEDIENTE No. _____

ZENITH ELECTRONICS
CORPORATIONS.

TITULO: ESTIMADOR DE CAWAL
- CONJUGADO y EFICIENTE
BASADO EN GRADACIONES

200.



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Industria y Comercio

SUPERINTENDENCIA

Delegatura de Propiedad Industrial División de Nuevas Creaciones

REQUISITOS NECESARIOS PARA PRESENTACIÓN Y AGILIZACIÓN DEL TRAMITE

PATENTE DE INVENCION ()	MODELO DE UTILIDAD ()	USUARIO ()	RADICADOR ()
-Indicación de que se solicita la concesión de una patente.	()	()	()
-Datos de identificación del solicitante o de la persona que se presenta la solicitud.	()	()	()
-Descripción de la invención.	()	()	()
-Dibujos de ser estos pertinentes.	()	()	()
-Comprobante de Pago de las tasas establecidas.	()	()	()

COMPLETA () INCOMPLETA () () ()

DISEÑO INDUSTRIAL ()

-Indicación de que se solicita el registro de Diseño Industrial.	()	()
-Datos de identificación del solicitante o de la persona que se presenta la solicitud.	()	()
-Representación gráfica y fotográfica del Diseño Industrial o muestra del Material que incorpora el diseño.	()	()
-Comprobante de Pago de las tasas establecidas.	()	()

COMPLETA () INCOMPLETA () () ()

ESQUEMA DE TRAZADO ()

-Indicación de que se solicita el registro de Diseño Industrial.	()	()
-Datos de identificación del solicitante o de la persona que se presenta la solicitud.	()	()
-Representación gráfica del Esquema Trazado.	()	()
-Comprobante de Pago de las tasas establecidas.	()	()

COMPLETA () INCOMPLETA () () ()

Firma Responsable:

Fecha:

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REPUBLICA DE COLOMBIA
SUPERINTENDENCIA DE INDUSTRIA Y COMERCIO

Bogotá,
2020

Doctor(a)
CARDENAS NAVAS DARIO
Apoderado(a) y/o Representante de
ZENITH ELECTRONICS CORPORATION

REFERENCIA	Radicación No.	6 54051
	Solicitud internacional	US2004/034642
	Fecha inicio fase nacional	05/07/2006
	Trámite	11
	Evento	379
	Actuación	430
	Oficio No.	

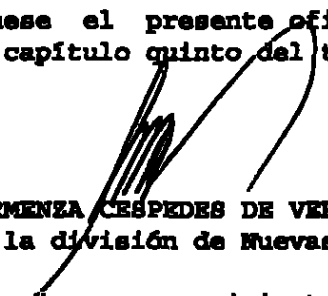
06725

Como resultado del examen de forma, realizado con fundamento en el artículo 38 de la Decisión 486 de la Comisión de la Comunidad Andina, artículo 27 del Tratado de Cooperación en Materia de Patentes (PCT), regla 51 bis del Reglamento y Circular Única de la Superintendencia de Industria y Comercio, se le comunica que:

1. Si fuere el caso, el solicitante debe adjuntar la traducción de los anexos del reporte de examen preliminar internacional (Art. 36.2.b) del tratado PCT.

En cumplimiento de lo dispuesto por el artículo 39 de la Decisión 486, se le requiere para que dentro del término de dos meses siguientes a la fecha de notificación del presente oficio, complete los requisitos anteriormente enunciados. En caso de no dar cumplimiento al presente requerimiento, la solicitud se considerará abandonada y perderá su prelación.

Notifíquese el presente oficio conforme a lo dispuesto en el numeral 5.2, literal e), del capítulo quinto del título primero de la Circular Única No.10 de 2001.


ALIX CARMONA CESPEDES DE VERGEL
Jefe de la división de Nuevas Creaciones

El anterior requerimiento fue notificado por fijación en lista de
fecha 23 JUN 2006
adpall