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SUPERINTENDENCIA DE INDUSTRIA Y COMERCIO



No. 09-059245- -00006-0000

Fecha: 2012-01-24 11:58:30 Dep. 2020 DIR.NUEVASCR
Tra. 11 PATENTEIYII Eve: 379 FASENACIONALII
Act. 523 RESPUESTA Folios: 10

Señor Director
DIRECCIÓN DE NUEVAS CREACIONES
Superintendencia de Industria y Comercio
E. S. D.

Trámite : 011
Evento : 379
Actuación : 444

Referencia : Expediente No. 09059245
Asunto : Solicitud de patente para **MÉTODO Y SISTEMA PARA
DETECTAR FUENTES DE SEÑAL EN UN ESPACIO DE
VIGILANCIA**
Solicitante : ELTA SYSTEMS LTD
Clase : G01S 003/048, G01S 003/074
Fecha solicitud : 08 de junio de 2009

1. Yo, Luz Clemencia DE PÁEZ, en mi condición de apoderada de la sociedad solicitante de la patente de la referencia, me refiero al oficio No. 94 notificado por fijación en lista del 05 de enero de 2012.

2. Mediante oficio No. 94 el Examinador a cargo del estudio de patentabilidad de la presente solicitud, en virtud del artículo 46 de la Decisión 486, solicitó la presentación de los siguientes documentos:

2.1 CHAN S C ET AL: "Efficient implementation of wideband multibeam forming network using sopot coefficients and multiplier block" DIGITAL SIGNAL PROCESSING, 2002. DSP 2002. 2002 14TH INTERNATIONAL CONFERENCE OF SANTORINI, GREECE 1-3 JULY 2002, PISCATAWAY, NJ, USAJEEE, US, vol. 1, 1 July 2002 (2002-07-01), pages 243-246, XP010599723 ISBN: 0-7803-7503-3.

2.2 LIE S ET AL: "Wideband polynomial-phase parameter estimation in sensor array" SIGNAL PROCESSING AND INFORMATION TECHNOLOGY, 2003. ISSPIT 2003. PROCEEDINGS OF THE 3RD IEEE INTERNATIONAL SYMPOSIUM ON DARMSTADT, GERMANY 14-17 DEC. 2003 PISCATAWAY, NJ, USAJEEE, 14 December 2003 (2003-12-14), pages 314-317, XP010729157 ISBN:0-7803-8292-7

3. Respuesta al requerimiento

Con el ánimo de dar respuesta al requerimiento formulado por el Examinador, se presenta junto con el presente memorial, copia de los dos documentos arriba mencionados.

4. Puesto que se han atendido a cabalidad las observaciones hechas por su Despacho sobre la presente solicitud de patente, atentamente solicito que se continúe su tramitación.

5. Anexos

- Documento: CHAN S C ET AL: "Efficient implementation of wideband multibeam forming network using sopot coefficients and multiplier block"
- Documento: LIE S ET AL: "Wideband polynomial-phase parameter estimation in sensor array"

Señor Director,


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COLO-16205-841-003-4
JMS\JMS\ID-209116\WPC16205
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EFFICIENT IMPLEMENTATION OF WIDEBAND MULTIBEAM FORMING NETWORK USING SOPOT COEFFICIENTS AND MULTIPLIER BLOCK

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ABSTRACT

This paper proposes an efficient implementation structure for the wideband multi-beam forming network in beamspace adaptive array for wideband signals. The filter coefficients of the beamforming network are represented as sum-of-powers-of-two coefficients (SOPOT) and are implemented using the multiplier block technique. The SOPOT representation allows the coefficients to be implemented as limited number of adders and shifters without any multiplications. The redundancy in the SOPOT coefficients are further removed by the multiplier block technique to yield minimum adder realization. Design results showed that significant reduction in hardware complexity can be obtained by the proposed structure as compared with conventional direct form implementation.

I. INTRODUCTION

Wideband beamforming using sensor arrays is an effective method for suppressing interference whose angles of arrival are different from the desired looking direction. They find important applications in radio communications, sonar, radar, and acoustics [1-3]. Traditional adaptive wideband beamformer usually employs tapped-delay line with adaptive coefficients to generate appropriate beam patterns for interference suppression. This usually requires considerable number of adaptive coefficients resulting in rather long convergence time and high implementation complexity. Several methods including the use of adaptive infinite impulse response filter (IIR) [12] and beamspace adaptive arrays [4,5] were proposed to overcome these problems. For the beamspace adaptive arrays in [5], interference suppression is achieved by combining the outputs from a fixed beamforming network which generates a set of beams pointing in different directions (Fig. 1). Since the number of beams selected is usually small, the number of adaptive weights for combining them are greatly reduced, resulting in faster convergence and fewer number of high speed variable multipliers. Although the number of variable multiplier is significantly reduced, the fixed beamforming network does require large number of fixed multipliers. Moreover, the complexity increases with the number of antenna, number of beams and the spatial-temporal selectivity of the fan filters in the beamforming network. In this paper, a novel implementation of the fixed beamforming network using canonical signed digits (CSD) representation (or sum-of-powers-of-two coefficients (SOPOT)) of the filter coefficients and a technique called multiplier block (MB) [11] is introduced. More precisely, the coefficients of the beamforming network are represented in CSD which requires only limited number of additions and hardware shifts for implementation. In addition, the number of additions to realize multiplications with the CSD coefficients can further be reduced by implementing the multiplications as a multiplier block. Multiplier block is an efficient method for performing the multiplications of a multiplicand with a set of fixed constant multipliers, by sharing subexpressions or immediate results common to the multipliers. In principle, it is possible to remove the redundancies in the constant multiplications leading to a minimum-adder-graph [11] with minimum adder realization. Design results show that the implementation complexity of the fixed beamforming network can be reduced drastically by the proposed technique. For example, with N sensors and L beams in the multibeam forming network each with a length- M FIR filter, the total number of multipliers required is LMN . For $L=5, N=15, M=15$, direct implementation will require $L \cdot M \cdot N = 1125$ multipliers. On the other hand, if the coefficients are represented as SOPOT coefficients with 8-bit

accuracy, then the proposed structure requires only 53 adders with an admirable frequency response. It should be noted that the proposed method is also applicable to the frequency invariance beamformer (FIB) in [6,7] and the infinite impulse response (IIR) multibeam former in [8][9], which approximates the fixed finite impulse response (FIR) filters in the multibeam former network by IIR filters. The complexity is reduced by a factor of 56.9% for $L=12, N=31, M=51$. However, the implementation of the IIR filters is somewhat complicated due to their finite wordlength effect. In addition, the redundancy in the filter coefficients are not fully explored and the overall complexity still grows rapidly with the number of taps, beams and antennas in the multibeam network. Although the multiplier block technique can also be used to realize efficiently these IIR filters, we only consider the FIR case because of its simplicity mentioned above and the large amount of redundancy existing in the coefficients as we shall see later in Section IV.

This paper is organized as follows: in section II, the principle of the beamspace adaptive beamformer and the design of the fixed beamforming network are described. Section III is devoted to the proposed implementation of the beamforming network. This is followed by the design results in Section IV. Finally, conclusions are drawn in section V.

II. BEAMSPACE ADAPTIVE BEAMFORMER USING FAN FILTER

Fig. 1 shows the structure of the beamspace adaptive array for wideband signals proposed in [5]. The digital received signal received from the sensors, after analogue-to-digital conversion and possibly downconversion, are fed into a wideband multibeam forming network, which is a set of spatial filters with fixed directions and hence coefficients. If $x_k(n)$, $k=0, \dots, N-1$, is the received signal from the k^{th} sensor, the output of the l^{th} beam forming network is

$$y_l(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{N-1} h_{l,k}(m) x_k(n-m) \quad (1)$$

where $h_{l,k}(m)$ is the impulse response of the l^{th} beamforming network. Its frequency response [5] as a function of f and θ is given by

$$H_l(f, \theta) = \sum_{k=0}^{N-1} \sum_{m=0}^{M-1} h_{l,k}(m) \cdot e^{-j2\pi f m T} \cdot e^{j2\pi f d \sin \theta \cdot k} \quad (2)$$

$$l=1, 2, \dots, L$$

where T is the sampling period, f is the frequency, θ is the angle of arrival (AOA) or direction of arrival (DOA) measured from the normal of the linear array, d is the antenna spacing and c is the propagation speed of the receiving signal. The

number of sensors is N and the number of taps per beam is M . By introducing the frequency variables f_1 and f_2 as follows

$$f_1 = f \cdot T_s \text{ and } f_2 = f \cdot d \sin(\theta) / c. \quad (3)$$

(2) can be rewritten in the familiar form of a 2D digital filter

$$H_i(f, \theta) = \hat{H}_i(f_1, f_2) = \sum_{m=0}^{M-1} \sum_{n=0}^{M-1} h_{i,k}(m) \cdot e^{-j2\pi m f_1} \cdot e^{j2\pi n f_2}. \quad (4)$$

To approximate a flat unit response for all frequencies f at a fixed direction θ_i (either for receiving the desired signal or suppressing the interference by subtracting it from the main beam), the ideal response of $H_i(f, \theta)$, up to a certain delay, should be

$$H_i(f, \theta_i) = \delta(\theta - \theta_i) = \hat{H}_i(f_1, f_2), \quad (5)$$

where $\delta(\theta)$ is the delta function. (5) suggests that the ideal response of $H_i(f, \theta_i)$ in the (f_1, f_2) -plane, i.e. $\hat{H}_i(f_1, f_2)$, should be a line with unit amplitude as given by

$$f_2 = f_1 \cdot d \sin(\theta_i) / (cT_s), \quad (6)$$

according to (3). In practice, the array spacing of a linear array d is usually chosen as $\lambda/2$ to avoid undersampling of the receiving signal and too much coupling between the antennas. Fig. 2 shows such a line at angle θ_i with $d = \lambda/2 = cT_s$. In practice, $H_i(f, \theta_i)$ is approximated as a set of fan filters with passband given by the shaded area in Fig. 2. The outputs from the L beams of the multibeam forming network, which represents received signal in directions θ_i , $i=1, \dots, L$, are adaptively selected and combined using the LMS algorithm to form an estimate of the interfering signals to be subtracted from the main beam (beam 0 with $\theta_0 = 0^\circ$ is assumed to be the looking direction). The coefficients of the fan filter can be determined by the transformation and windowing method proposed in [5], which is also used in this work. More precisely, a one dimension prototype filter with desired bandpass response and zeros at the center frequencies of other beams is first designed. It is then transformed by means of the transformation to obtain the frequency response of the fan filter. The frequency response of the fan filter in the grey area of Fig. 2 is set to zero, though it can be chosen arbitrary. An inverse discrete fourier transform is then applied to the sampled frequency response to obtain the desired filter coefficients. This 2-D impulse response is then windowed to obtain the final filter coefficients.

III. NOVEL IMPLEMENTATION OF THE MULTIBEAM FORMING NETWORK

It can be seen from (1) that the beam forming network for the i^{th} beam is actually obtained by summing the outputs of a set of FIR filters, each with its input coming from one sensor element in the sensor array. To see this, rewrite (1) as follows

$$y_i(n) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} h_{i,k}(m) x_k(n-m) = \sum_{m=0}^{M-1} g_{i,k}(n) \quad (7)$$

where $g_{i,k}(n) = \sum_{m=0}^{M-1} h_{i,k}(m) x_k(n-m)$ is the output of the FIR filter with impulse response $h_{i,k}(m)$ and its input is derived from the k^{th} sensor element (see Fig. 3 for the overall structure).

To achieve a multiplier-less realization of the multibeam forming network, let's replace the coefficients $h_{i,k}(m)$ in the FIR filters as the following sum of powers-of-two coefficients (SOPOT) or canonical signed digits (CSD).

$$b = \sum_{i=0}^{I-1} a_i \cdot 2^{-i}, \text{ where } a_i \text{ is either } 1 \text{ or } -1, \text{ and}$$

$$b_i \in \{-l_L, \dots, -1, 0, \dots, l_U\}. \text{ The larger the numbers } l_L, l_U, \text{ and } L,$$

the closer the SOPOT approximation will be to the original real number. In practice, the number of non-zero terms is usually kept to a small number while satisfying a given specification so that the multiplication can be implemented by a limited number of shift and add (subtract) operations, giving rise to multiplier-less realization. As mentioned earlier, the number of adders required to implement $h_{i,k}(m)$ in SOPOT representation can further be reduced using the technique of multiplier block. The idea is to rewrite $h_{i,k}(m)$ in transposed form for computing

$$g_{i,k}(n) = \sum_{m=0}^{M-1} h_{i,k}(m) x_k(n-m) \text{ as shown in Fig. 3. It can be}$$

seen that instead of multiplying the delayed input samples with the filter coefficients as in the direct form, the input sample is now multiplied with all the coefficients. This can be efficiently implemented using a multiplier block (MB) [9]. Let's consider a simple example with two filter coefficients: 3 and 21. The SOPOT representations of these two numbers are: $3 = 2^1 + 1$ and $21 = 2^4 + 2^2 + 1$. This requires 3 adders and 3 shifts. If implemented in a MB, the multiplication of the input with the coefficient 3 will also be generated by decomposing 3 as $2^1 + 1$, requiring one addition. The multiplication with 21, however, can be simplified by re-using the intermediate result generated by the first filter coefficient '3' as $21 = 3 \cdot 7 = 3 \cdot (2^3 - 1)$. Actually, the intermediate result, after multiplication by 3, is multiplied by 7, which requires one less adder than generating 21 directly. In principle, it is possible to remove all the redundancy found in the constant multipliers leading to a realization with the minimum number of adders. This can drastically reduce the number of adders required for realizing the multibeam forming network, as will be illustrated later in Section IV. A further saving can be obtained by grouping $h_{i,k}(m)$ with a fixed value of k and implement them as one multiplier block (MB#0, to MB#N-1 in Fig. 3). Therefore all the redundancy in $h_{i,k}(m)$, $i=1, \dots, L$; $m=0, \dots, M-1$, for a fixed value of k can be removed, yielding a minimum adder realization. The generation of the MB follows the method of Dempster *et al.* proposed in [11]. Interested readers are referred to [11] for more details. Next, we consider a design example.

IV. DESIGN EXAMPLE

For comparison, we consider the same example designed in [5]. The number of antenna elements is 15. All the digital FIR filters has 15 taps. The multibeam forming network consists of 5 beams pointing at angles $0^\circ, \pm 24^\circ$ and $\pm 53^\circ$, respectively. Figure 4 shows their frequency responses when the filter coefficients are represented in CSD with 8-bit accuracy (as the frequency responses at the angles $\pm 53^\circ$ and $\pm 24^\circ$ are symmetric, only the beam patterns at $-53^\circ, -24^\circ$ and 0° are shown). Due to page limitation, the frequency responses with different wordlength for the filter coefficients are omitted here. It was found that the 8-bit SOPOT frequency response is comparable even with the full precision results. Table I compares the hardware complexity of using the conventional method, the proposed SOPOT coefficient method, and the proposed SOPOT-MB method in implementing the multibeam forming network under consideration. The conventional method requires 1125 multipliers, (excluding the 1050 additions for implementing the delay line additions required in the FIR filters). The proposed SOPOT realization reduces the implementation complexity to 350 adders for 8-bit accuracy and 3140 adders for 16-bit accuracy. Using the multiplier block technique, the number of adders goes down to 53 for 8-bit accuracy and 922 for 16-bit accuracy. An order of magnitude improvement in hardware complexity is achieved by the proposed method. It can therefore be concluded that the proposed method can

drastically reduce the hardware complexity in implementing the multibeam forming network for the beamspace wideband adaptive beamformer.

V. CONCLUSION

A novel and hardware efficient structure for implementing the wideband multibeam network for beamspace adaptive array is presented. The proposed structure, which is based on the SOPOT representation of the filter coefficients and the multiplier block technique, significantly reduces the hardware complexity compared with conventional direct form implementation.

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	Direct Implementation	
No. of mul.	15×15×5=1125	
	No. of Adders	
No. of bits/ coefficient	Proposed structure SOPOT only	Proposed structure SOPOT and MB
8	350	53
9	582	88
10	1001	146
11	1297	234
12	1703	361
13	2033	487
14	2410	668
15	2772	792
16	3140	922

Table 1. Hardware complexity of various methods. N=15, M=15, L=5. (Only consider all the FIR filters in the BFNs, excluding the delay line addition.)

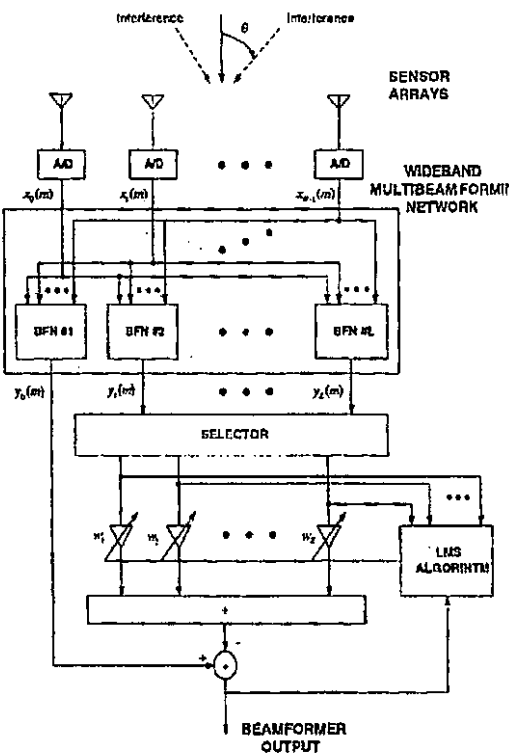


Figure 1. Beamspace adaptive array for wideband signals. Beamforming network (BFN).

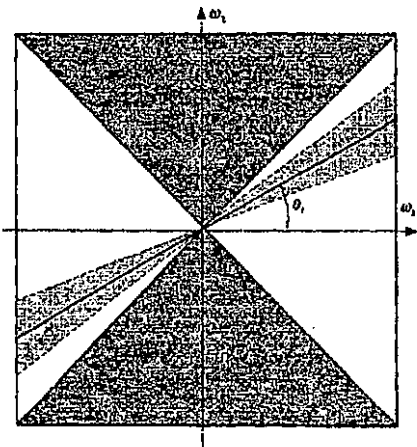


Figure 2. Frequency response of the ideal fan filter.

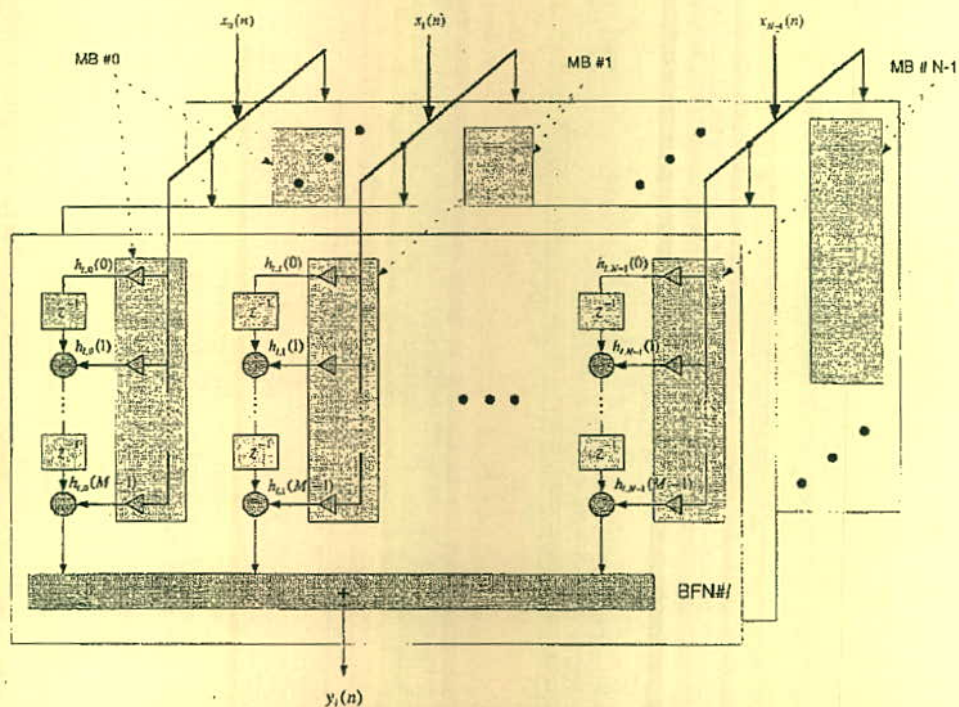


Figure 3. Proposed multibeam forming network using SOPOT coefficients and multiplier block (MB).

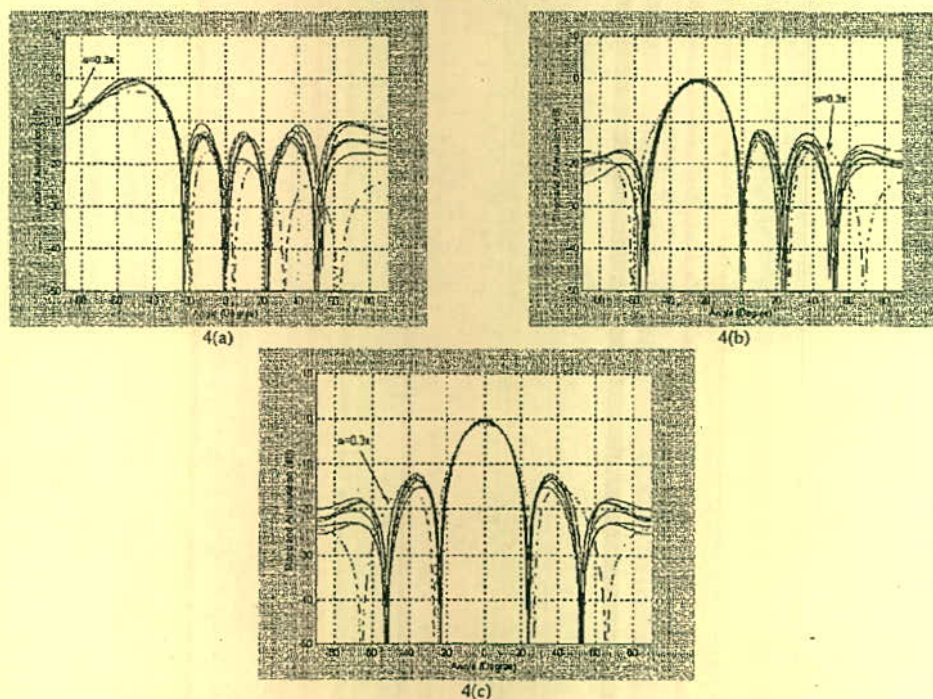


Figure 4. Frequency response of the multibeam forming network at seven frequencies: $0.3\pi, \dots, 0.9\pi$ (a) -53° , (b) -24° , (c) 0° .

Wideband Polynomial-Phase Parameter Estimation In Sensor Array

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Abstract— This paper proposes a closed-form estimation technique for a wideband constant amplitude polynomial-phase signal embedded in white Gaussian noise. It is shown that it is possible to jointly estimate the highest order phase coefficient and direction of arrivals of the signal through a closed-form method. Hence it is, computationally, much simpler than Maximum Likelihood method which involves highly non-linear multidimensional optimization. Simulation and comparative results are included to show its effectiveness.

I. INTRODUCTION

The subject of this paper is the parameter estimation of a Polynomial-Phase Source (PPS) signal in sensor arrays. The parameter estimation of PPS signals is an important problem because they arise in a wide-range of practical radar and communication applications. There is an extensive literature on the parameter estimation of PPS methods mainly suboptimal methods, because the optimal Maximum Likelihood (ML) method is computationally expensive. For single sensor system example, in [7], Discrete Polynomial phase Transform was employed to estimate the parameters by means of multiple 1-D searches through FFT. Sensor arrays PPS have been featured in many works on parameter estimation, notably, algorithms in [1], [2] and [5] are important contribution. In [6], a subspace-based method was introduced to estimate frequency and the frequency rate using only a single sensor and relying on the special structure of time-sampled correlation matrix. This method cannot be extended to estimation of Direction Of Arrival (DOA) together with the said parameters because it requires the same special structure inherent in the spatial-sampled correlation matrix. In general other conventional subspace based DOA estimators also

cannot be applied directly due to non-stationarity of the true correlation matrix of array data vector. In [1] the wideband technique called chirp beamformer(CBF) was devised to extract the parameters of interest by means of a 3-D search.

In this paper, we describe a much simpler search-free parameter estimation technique of wideband PPS signal, primarily to estimate the highest order phase coefficient and DOA. We exploit the dual shift-invariance property that appears after a preprocessing step similar to the step before FFT in [7] and post process with ESPRIT-based joint angle-frequency estimation method [3]. The proposed technique is effective and cheap. Simulation experiments and comparative results are included to validate the strength of the proposed algorithm.

II. SIGNAL MODEL

The signal model for a constant amplitude PPS signal impinging at a uniform linear array (ULA) antenna composed of M sensors can be described as:

$$\mathbf{x}(n) = \mathbf{a}(\theta, n)s(n) + \mathbf{v}(n), \quad n=0, \dots, N-1 \quad (1)$$

where $\mathbf{a}(\theta, n)$ is the $M \times 1$ array steering vector, $s(n)$ is the PPS signal waveform and $\mathbf{v}(n)$ is the $M \times 1$ vector of circularly complex Gaussian noise that is zero mean, temporally and spatially white, and N is the number of snapshots. The PPS signal waveform of order K is modeled as $s(n) = A \exp(j \sum_{k=0}^K a_k (n\Delta)^k)$, where A is the unknown constant amplitude, $a_k = 2\pi f_k$ is the k^{th} order coefficient and Δ is the sampling interval. In addition to that, the array steering vector is modeled as $\mathbf{a}(\theta, n) = [1, e^{j\alpha(n)\psi}, \dots, e^{j\alpha(n)\psi(M-1)}]^T$, where the instantaneous frequency is modeled as $\alpha(n) = \sum_{k=0}^{K-1} (k+1)a_{k+1}(n\Delta)^k$, which is assumed to be constant during

the time necessary for propagation across the array aperture. The DOA, θ , is related to ψ by $\psi = \frac{d}{c} \sin(\theta)$, where d is the spacing between adjacent sensors of the ULA and c is the propagation speed in the medium.

III. PPS ESTIMATION

A. Estimation Algorithm

In order to vividly observe the main idea of the algorithm, we begin by rearranging (1) as follows,

$$(x(n))_m = A \exp\{j(a_K(n\Delta))^K + \sum_{k=0}^{K-1} (a_k + (k+1)a_{k+1}\psi(m-1))(n\Delta)^k\} \quad (2)$$

where $(x(n))_m$ is the m^{th} component of $x(n)$. We define an operator, $\text{DP}_K[x(n), \tau] \triangleq [\text{DP}_K[(x(n))_1, \tau], \dots, \text{DP}_K[(x(n))_M, \tau]]^T$, of order K^{th} , where $\text{DP}_K[\cdot]$ is the *high-order instantaneous moment* (HIM) operator defined in [4] and τ is *delay parameter*, which could be chosen according to [7] for time being. This operator basically operates at each sensor independently, hence it inherits the properties of $\text{DP}_K[\cdot]$ in temporal analysis. Thus, in case of no noise, and by applying the *Theorem 1* in [7] to (2), we obtain

$$y(n, \tau) = \text{DP}_K[x(n), \tau], \quad n = (K-1)\tau, \dots, N-1$$

where

$$(y(n, \tau))_m = B \exp\{j(\phi n + \mu m + \gamma)\} \quad (3)$$

$$\phi = K! \tau^{K-1} \Delta^K a_K \quad (4)$$

$$\mu = \phi \psi / \Delta \quad (5)$$

$$\gamma = (K-1)!(\tau \Delta)^{K-1} a_{K-1} - 0.5(K-1)K!(\tau \Delta)^K a_K$$

$$B = A^{2^{K-1}}$$

Hence, in compact form $y(n, \tau) = a(\mu) B e^{j(\phi n + \gamma)}$ where τ is hold constant¹ and $a(\mu)$ is just the narrowband array steering vector, i.e. $a(\mu) \triangleq [1, e^{j\mu}, \dots, e^{j\mu(M-1)}]^T$. If we stack over different samples as $\tilde{Y} = [y(0), y(1), \dots, y(N-1)]$, then we can observe the shift invariant in two directions, i.e. along different columns and along different rows. By forming m -factor temporal smoothing as follow,

$$Y = \begin{bmatrix} y(0) & y(1) & \dots & y(N-m) \\ y(1) & y(2) & \dots & y(N-m+1) \\ \vdots & \vdots & & \vdots \\ y(m-1) & y(n) & \dots & y(N-1) \end{bmatrix} \quad (6)$$

¹we'll suppress the τ notation from here on to save space

we could get the dual shift invariant to appear in proper selection of different set of rows. The proper selection matrices are

$$J_{x,\phi} = [I_{m-1} \ 0_1] \otimes I_M$$

$$J_{y,\phi} = [0_1 \ I_{m-1}] \otimes I_M$$

$$J_{x,\mu} = I_m \otimes [I_{M-1} \ 0_1]$$

$$J_{y,\mu} = I_m \otimes [0_1 \ I_{M-1}]$$

which are to be left-multiplied to left-singular vector that correspond to the largest singular value of (6) to get the following sets of equations,

$$\begin{aligned} J_{x,\phi} u &= a' t & J_{x,\mu} u &= a'' t \\ J_{y,\phi} u &= a' e^{j\phi t} & J_{y,\mu} u &= a'' e^{j\mu t} \end{aligned}$$

where $Y = u s_1 v^H$, s_1 is the largest singular value, a' , a'' are vectors of $a(\mu)$ with some rows deleted and t is a scaling constant. Let $u_{x\phi} = J_{x,\phi} u$, $u_{y\phi} = J_{y,\phi} u$, $u_{x\mu} = J_{x,\mu} u$ and $u_{y\mu} = J_{y,\mu} u$, we can obtain

$$\begin{aligned} u_{x\phi}^\dagger u_{y\phi} &= e^{j\phi} \\ u_{x\mu}^\dagger u_{y\mu} &= e^{j\mu} \end{aligned} \quad (7)$$

Thus the $\text{DP}_K[\cdot]$ operator has essentially transformed a PPS array signal, which is non-stationary, into a stationary joint angle-frequency estimation (JAFE) problem [3], which is based on ESPRIT and hence it is search-free and computationally efficient. The algorithm could give the estimate of ϕ and μ jointly, then following that, the estimate of the K^{th} order frequency parameter and DOA can be extracted by the following relations,

$$\hat{a}_K = \frac{\hat{\phi}}{K! \tau^{K-1} \Delta^K} \quad (8)$$

$$\hat{\theta} = \arcsin\left(\frac{c \Delta \hat{\mu}}{d \hat{\phi}}\right) \quad (9)$$

For better estimation of ϕ and μ , spatial-temporal smoothing and forward-backward averaging could be done instead of temporal smoothing alone [3].

B. Analysis and Identifiability Condition

In addition to the identifiability condition in JAFE algorithm, we will show the conditions on Δ , τ , a_K and d/c needed for performance and identifiability of the algorithm.

Firstly, note that in the angle-frequency estimation model, DOA and frequency of interest are related to μ and ϕ , respectively and independently of each other. While in PPS problem, as seen in (8) and (9), the K^{th} order frequency parameter is related to ϕ independent of

μ , but DOA is related to both μ and ϕ . This creates problem to estimate DOA accurately. To observe the effect of the parameters on estimation accuracy, the first order perturbation approximation, and the mean-square error (MSE) of $\hat{\psi}$ are derived as follows,

$$\begin{aligned}\delta\psi &\approx \frac{\delta\mu\Delta}{\phi} - \frac{\mu\Delta\delta\phi}{\phi^2} \\ E(\delta\psi^2) &\approx \frac{\Delta^2}{\phi^2} E(\delta\mu^2) + \frac{\mu^2\Delta^2}{\phi^4} E(\delta\phi^2) \\ &= \frac{\Delta^2}{\phi^2} E(\delta\mu^2) + \frac{\psi^2}{\phi^2} E(\delta\phi^2) \quad (10)\end{aligned}$$

We have used (5) to get (10). By substituting (4) to (10) and using the fact of $\delta\theta \approx \frac{c\delta\psi}{d\cos\theta}$, we finally obtain

$$E(\delta\theta^2) \approx \frac{(\Delta c/d)^2 E(\delta\mu^2) + (\sin\theta)^2 E(\delta\phi^2)}{(K!\tau^{K-1}\Delta^K a_K \cos\theta)^2} \quad (11)$$

Similarly the MSE of $\hat{a}_K$ can be obtained from (4) as follows,

$$E(\delta a_K^2) \approx \frac{E(\delta\phi^2)}{(K!\tau^{K-1}\Delta^K)^2} \quad (12)$$

From (11), it can be observed that MSE of $\hat{\psi}$ is affected by MSE of $\hat{\phi}$. Increasing the ratio of Δ to d/c will reduce the effect of estimation error of $\hat{\phi}$. Furthermore, if we increase Δ , we are reducing the hardware requirement to sample fast and at the same time the MSE of DOA and a_K can be reduced.

Secondly, the conditions to estimate DOA and a_K unambiguously are $|\phi| < \pi$ and $|\mu| < \pi$. These conditions could be rewritten using (4) and (5) as follow,

$$\begin{aligned}K!\tau^{K-1}\Delta^{K-1} &< \frac{\pi}{|a_K|\Delta} \\ K!\tau^{K-1}\Delta^{K-1} &< \frac{\pi}{|a_K|(d/c)}\end{aligned}$$

These could be combined as one inequality,

$$K!\tau^{K-1}\Delta^{K-1} < \frac{\pi}{|a_K|} \min\left(\frac{c}{d}, \frac{1}{\Delta}\right) \quad (13)$$

This inequality upper bounds the choices the parameter Δ we can use.

Finally, it should be noted that the performance of DOA relies on the actual parameter a_K ; the larger a_K , which roughly corresponds to larger-bandwidth, the better the performance of DOA. As for smaller a_K , we might still be able to compensate the performance by increasing Δ , but it is not possible for very small a_K . In this case, it is preferable to use the usual narrow band DOA estimation method directly or after the removal of higher order a_k 's from the signal using the method in [7].

IV. SIMULATION AND DISCUSSION

A. Simulation Examples

In these simulation examples, we consider a 10 element ULA with inter-element spacing of $d = 1.5m$ and $c = 1500m/s$, which is propagation speed of sonar. In the first two figures, root mean square error (RMSE) of a Linear FM signal, i.e. $K = 2$, is computed through 100 Monte-Carlo runs. The signal parameters are $f_0 = 0$, $f_1 = 420Hz$, $f_2 = 100\frac{Hz}{s}$, $\theta = 40^\circ$ with sampling interval of $\Delta = 4 \cdot 10^{-3}s$, number of samples $N = 256$ and $\tau = 128$. In addition to that, the spatio-temporal smoothing of factor $m = 64$ and $L = 3$ together with forward-backward data extension have been incorporated in the algorithm. In Fig. 1, the RMSE of f_2 and DOA plotted against the SNR. As for the comparison, ML method, simulation and theoretical result of proposed method are shown on the same figure.

In the second figure (Fig. 2), the RMSE of both f_2 and θ are plotted against Δ , while SNR is held at 30dB, to show the dependency of RMSEs toward choices of Δ . Note that the simulation for $\Delta \gtrsim 4 \cdot 10^{-3}$ is not shown because it violates the ambiguity condition (13), and hence there will be significant RMSE. It can be clearly seen that the theoretical results plotted are very close to the simulation results.

In the last simulation, we showed that the algorithm works well for order $K = 3$. The signal parameters are $f_0 = 0$, $f_1 = 300Hz$, $f_2 = 100\frac{Hz}{s}$, $f_3 = 100\frac{Hz}{s^2}$, $\theta = 40^\circ$ with sampling interval of $\Delta = 4 \cdot 10^{-3}$, number of samples $N = 255$ and $\tau = 85$. The spatio-temporal smoothing factors are $m = 42$ and $L = 3$. The RMSE results are plotted against SNR in Fig. 3.

B. Discussion

The algorithm shows the possible application of ESPRIT-based JAFE algorithm in estimation of DOA and frequency parameter of wideband PPS. The algorithm presented here is primarily for single component PPS signal, while direct applying the algorithm to multi-component PPS signal will encounter the cross-term problem due to the non-linear operator $DP[\cdot]$. This problem is severe and has been an open problem in time-frequency research domain. Since the algorithm here applied to single source, the JAFE algorithm becomes very simple, and the joint diagonalization step is not necessary (see (7)). The effect of varying τ to the performance is more complicated to be analyzed because it effects the number of data of $y(n, \tau)$ available for JAFE algorithm and the choice of optimum temporal smoothing factor

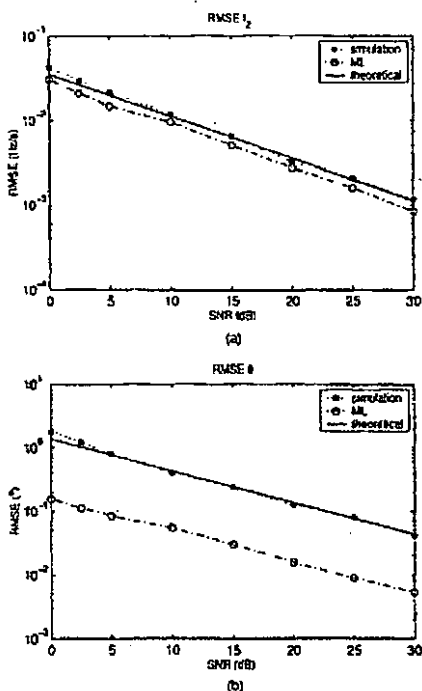


Fig. 1. Comparison of simulation results between ML and the proposed method. RMSE of f_2 (Hz/s) and DOA (°) as function of SNR are in (a) and (b) respectively

m , in addition to the inverse proportional relation in (11) and (12). Due to the lack of space, we will not discuss the effect of varying τ in this paper.

V. CONCLUSION

We have shown the application of joint-diagonalization algorithm to a single PPS signal. This method is a closed-form algorithm and hence computationally efficient. Further works are being done to extend the application of the algorithm to multiple sources PPS signal.

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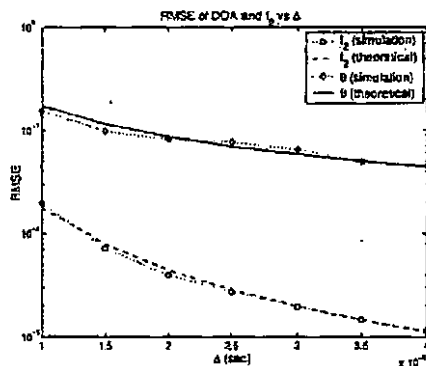


Fig. 2. RMSE of f_2 (Hz/s) and DOA (°) as function of Δ at SNR=30dB

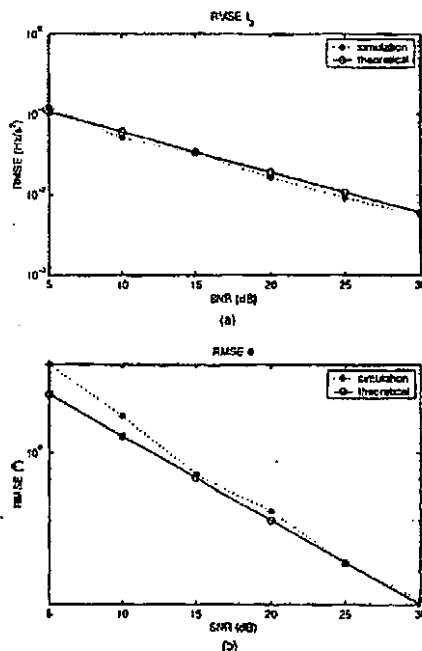


Fig. 3. RMSE of f_3 (Hz/s²) and DOA (°) as function of SNR are in (a) and (b) respectively